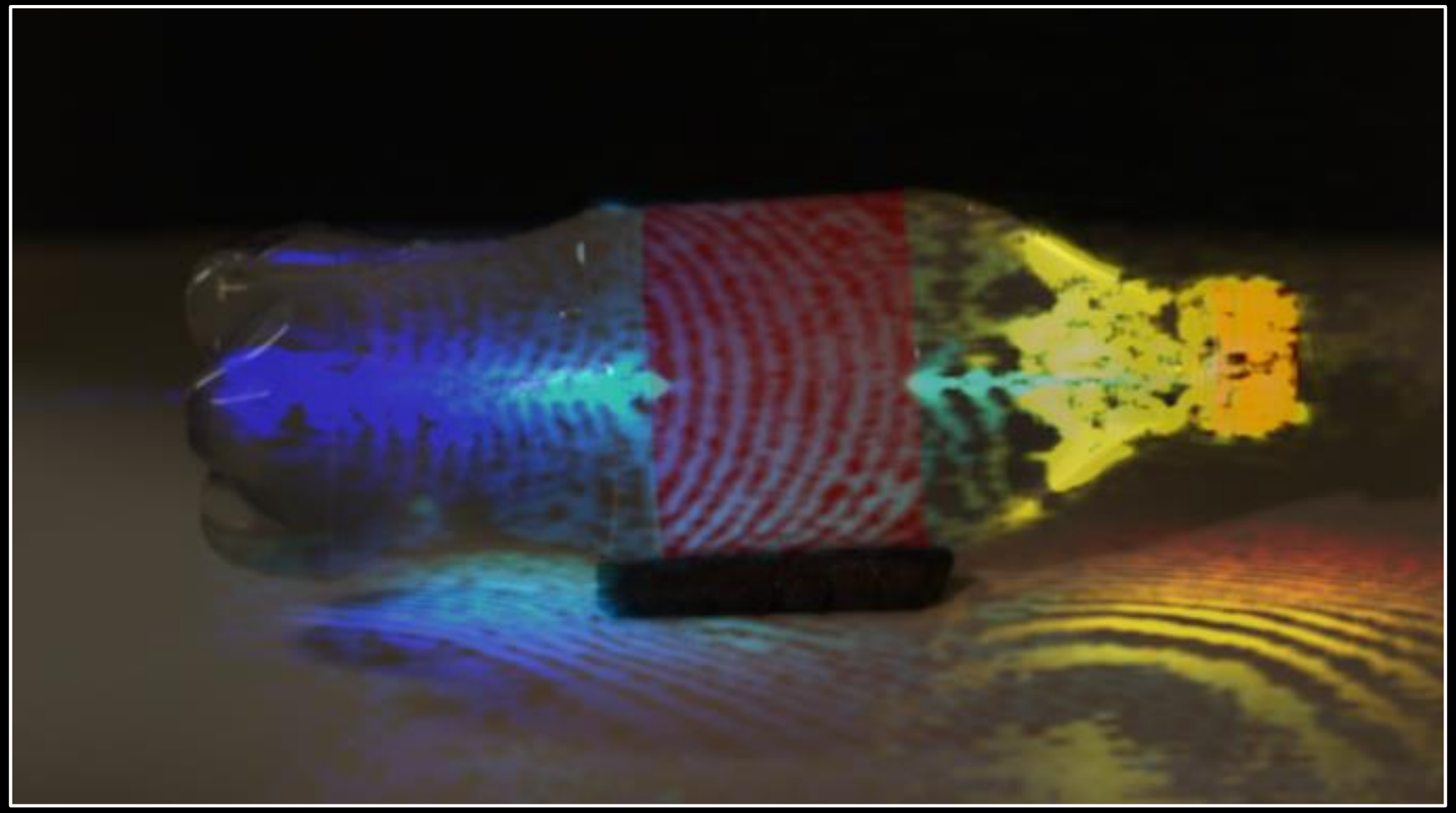
Synthetic Aperture Radar

Matthew O’Toole

Oct 29, 2026

From Light to Radar

transient imaging



speed of light in a vacuum: **299,792,458** meters / sec

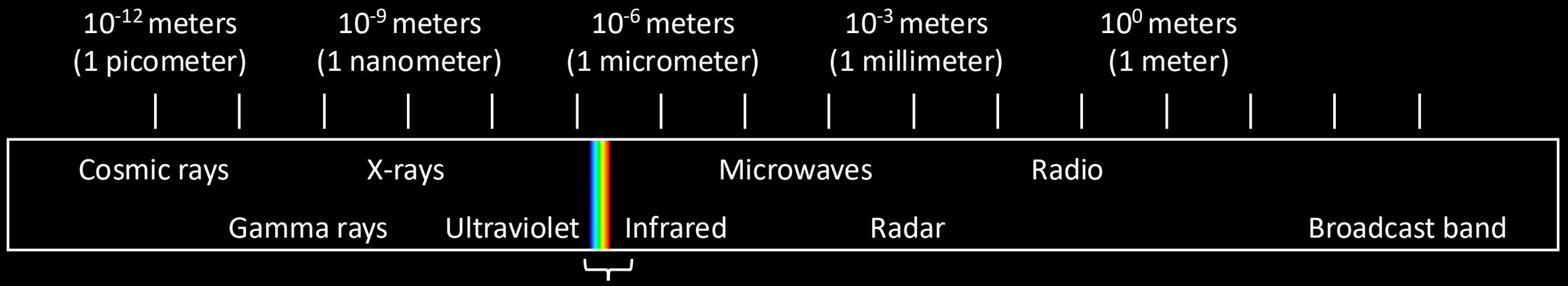
(Light travels approximation **1 MILLION** times faster than sound!)

transient imaging (a.k.a. femtophotography)



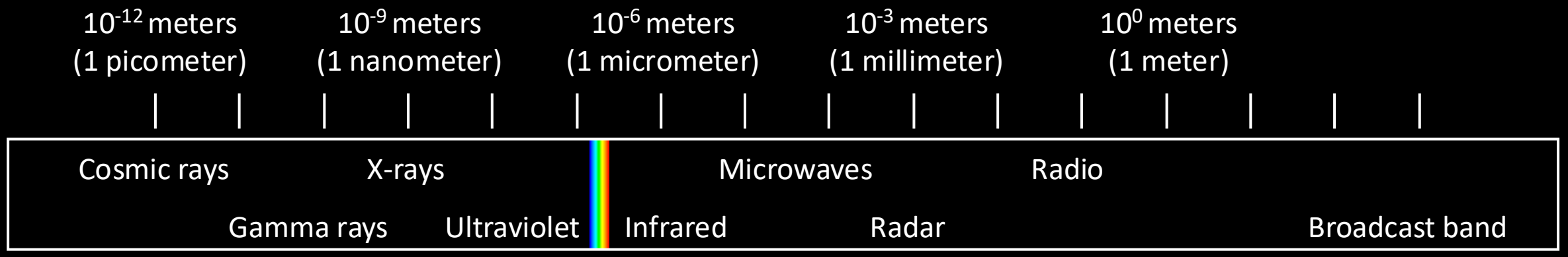
“Femto-photography: capturing and visualizing the propagation of light”, [Velten et al., 2012]

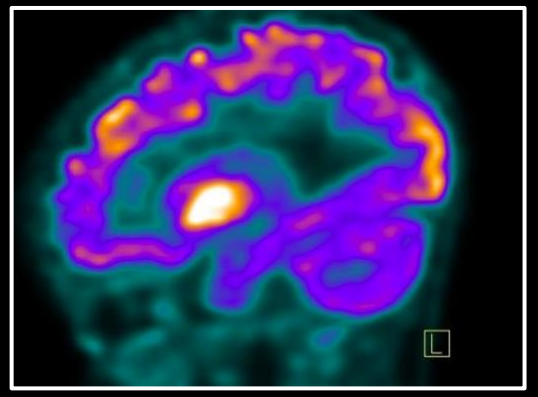
spectrum of transient sensing technologies



- **optical coherence tomography**
- **streak camera**
- **single-photon avalanche diodes**
- **time-of-flight cameras**
- **avalanche photodiode**

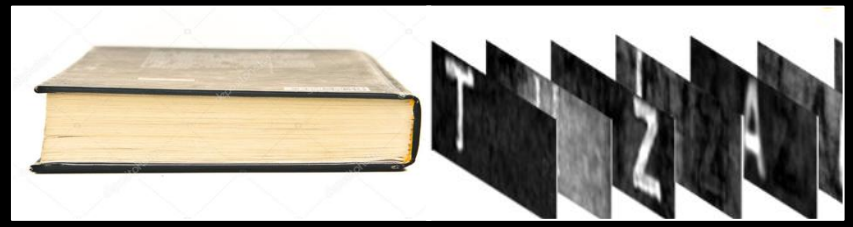
spectrum of transient sensing technologies



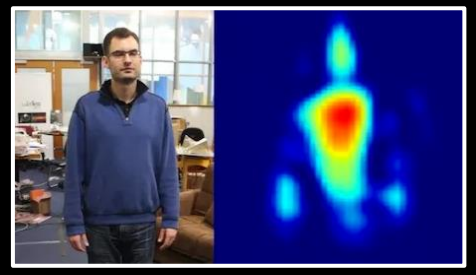


Positron Emission Tomography (PET) Scan

- optical coherence tomography
- streak camera
- single-photon avalanche diodes
- time-of-flight cameras
- avalanche photodiode



Terahertz Time-gated Imaging
[Redo-Sanchez et al., 2015]



Seeing through Walls
[Adib et al., 2015]

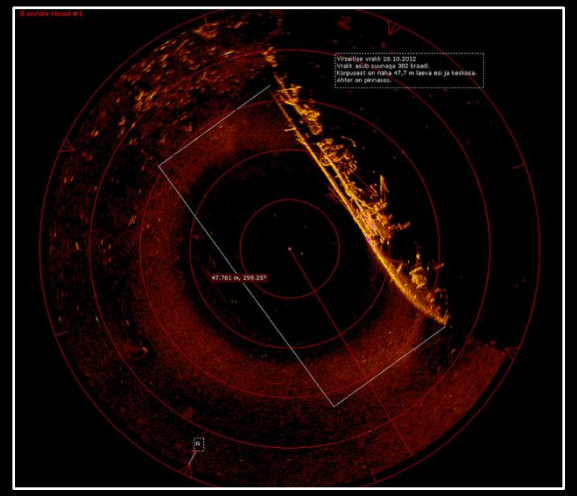
spectrum of transient sensing technologies

10^{-12} meters (1 picometer) | 10^{-9} meters (1 nanometer) | 10^{-6} meters (1 micrometer) | 10^{-3} meters (1 millimeter) | 10^{0} meters (1 meter)

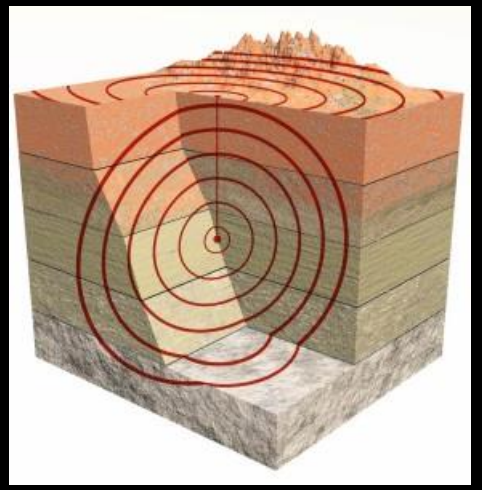
mechanical waves (e.g., acoustic, seismic)



Ultrasound Imaging

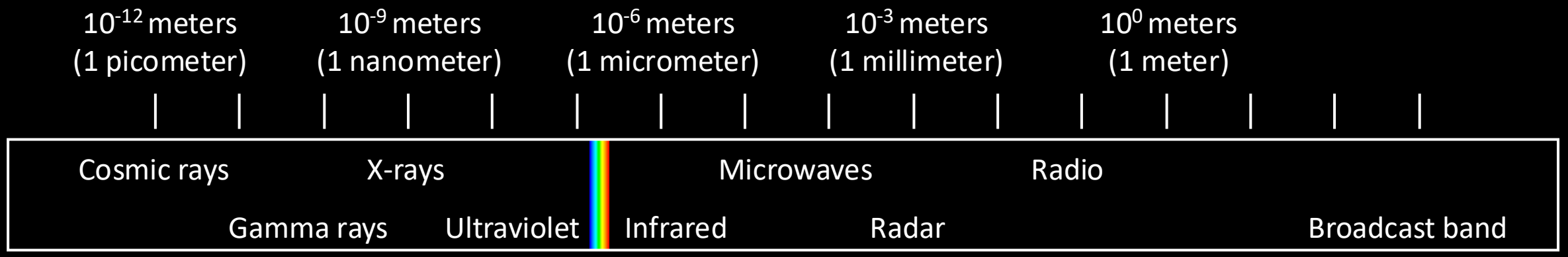


SONAR
(Sound Navigation and Ranging)

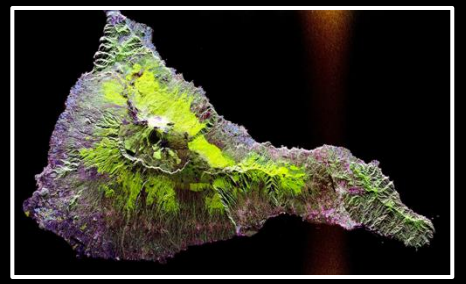


Seismic Imaging

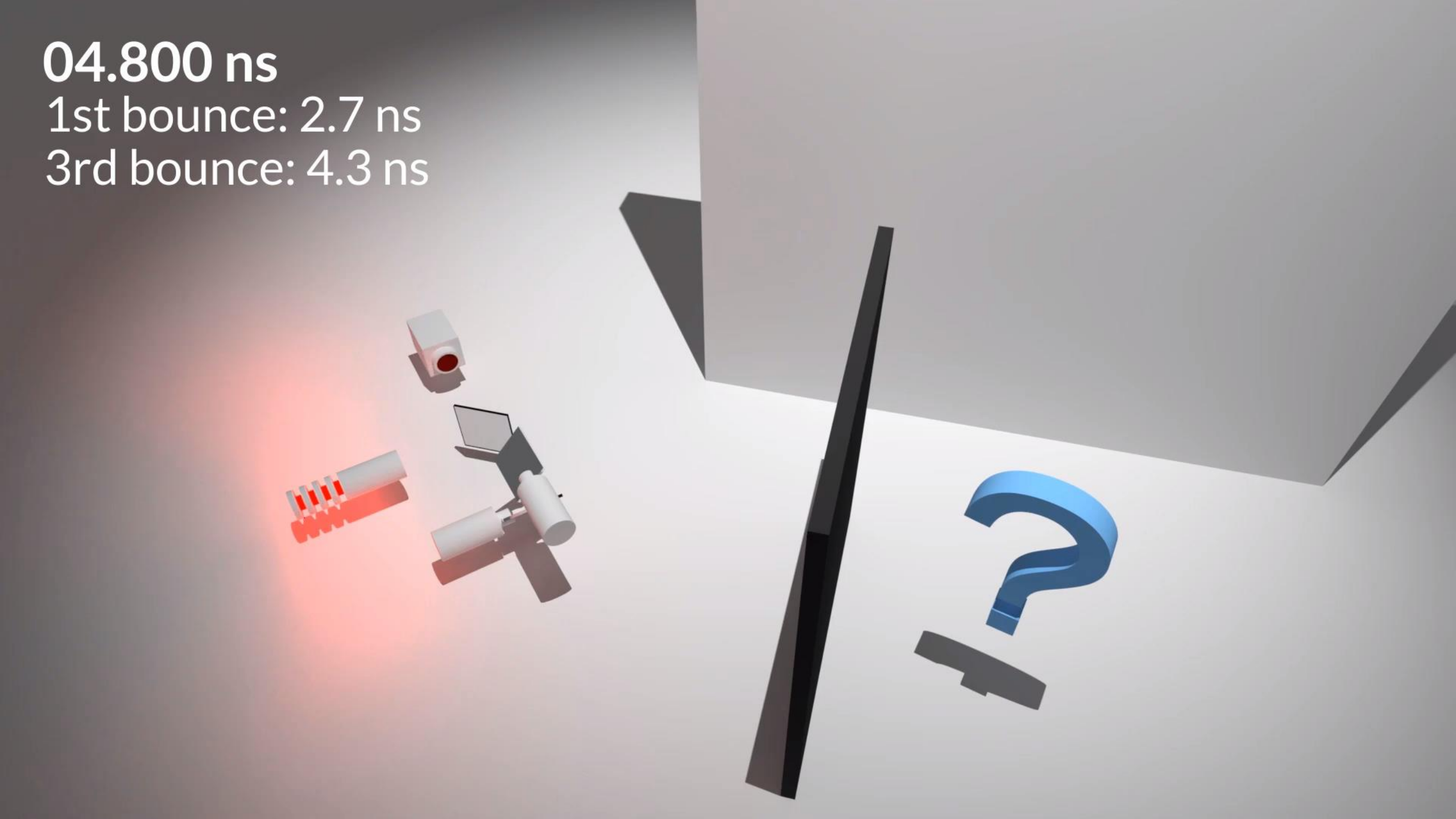
spectrum of transient sensing technologies

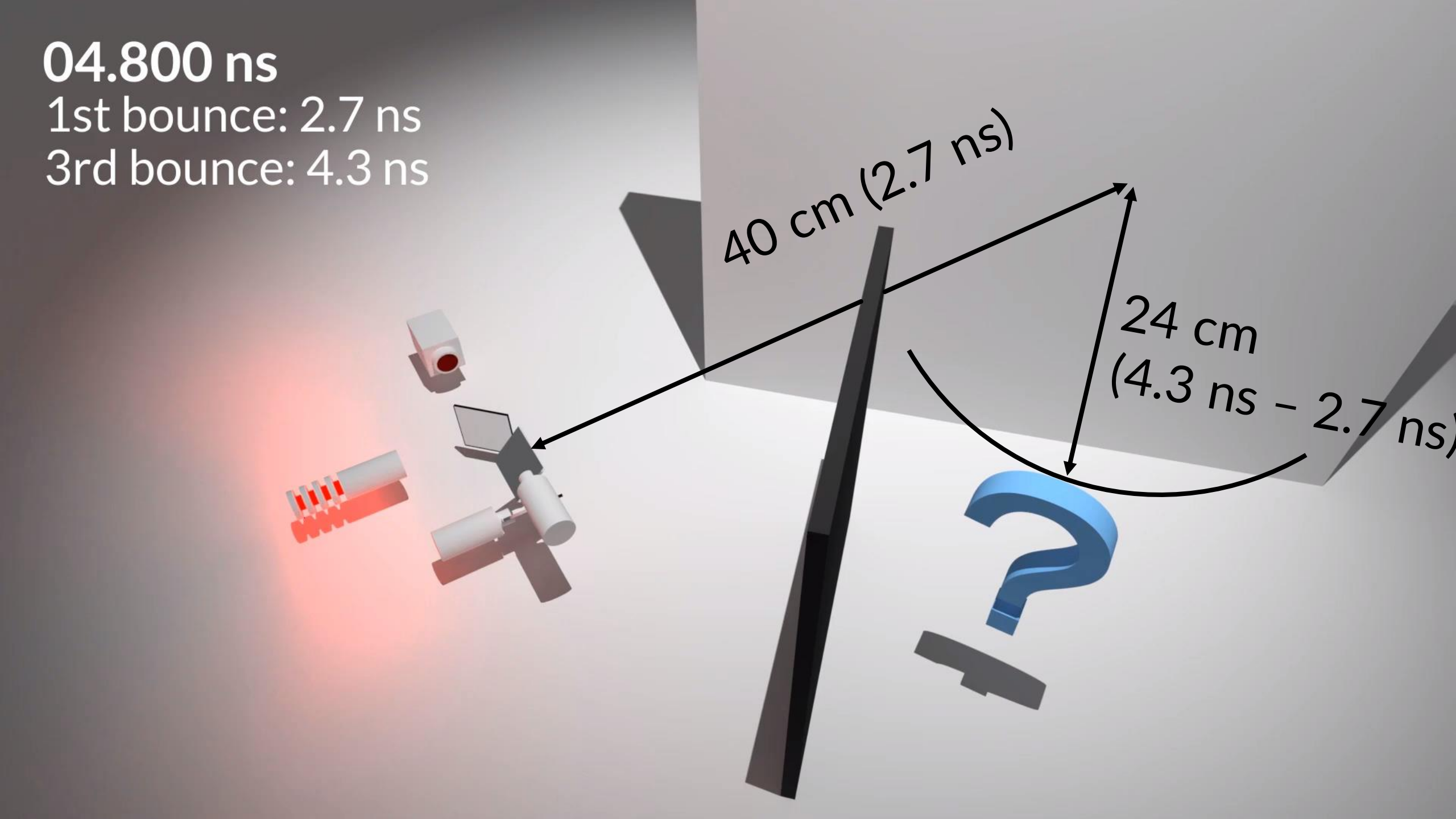


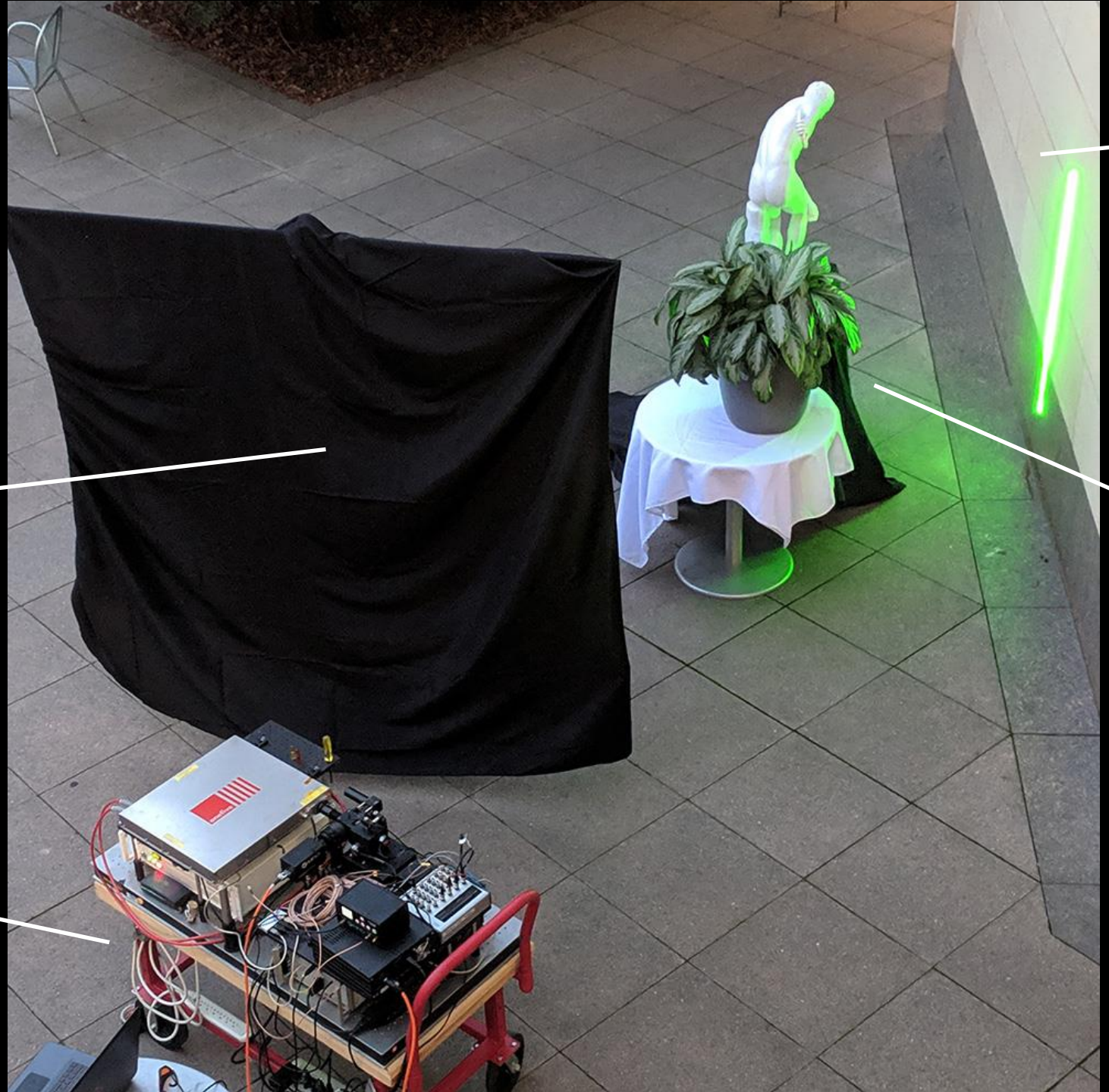
- optical coherence tomography
- streak camera
- single-photon avalanche diodes
- time-of-flight cameras
- avalanche photodiode



Synthetic Aperture Radar (SAR)


04.800 ns
1st bounce: 2.7 ns
3rd bounce: 4.3 ns


04.800 ns
1st bounce: 2.7 ns
3rd bounce: 4.3 ns
40 cm (2.7 ns)
24 cm
(4.3 ns – 2.7 ns)


wall
occluder
hidden scene
NLOS imaging system

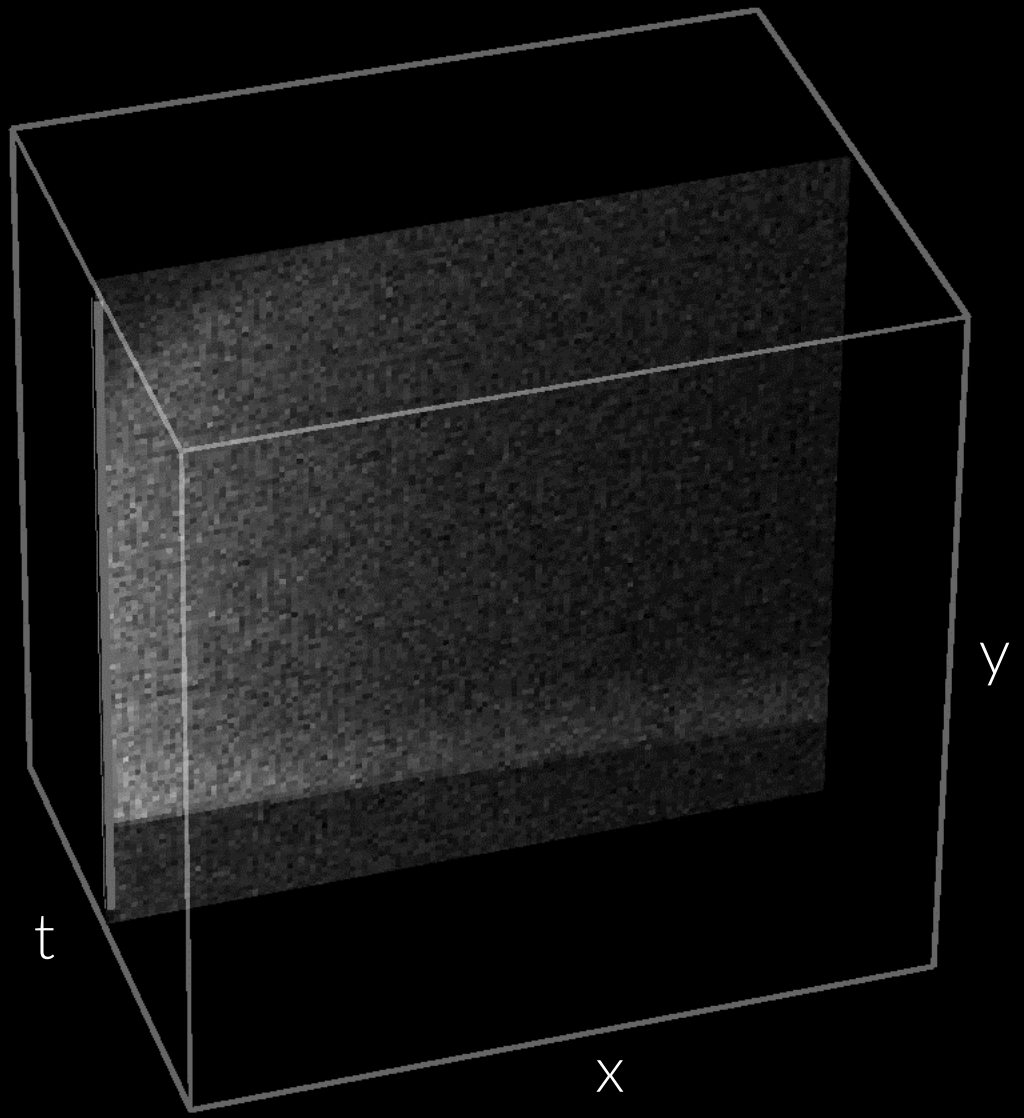

scanning mirrors
laser
detector
PICOSECOND
DELAYER
INPUT
MPD
NIM OUT
TTL OUT


resolution: 128 x 128
area: 2 m × 2 m

scene photo



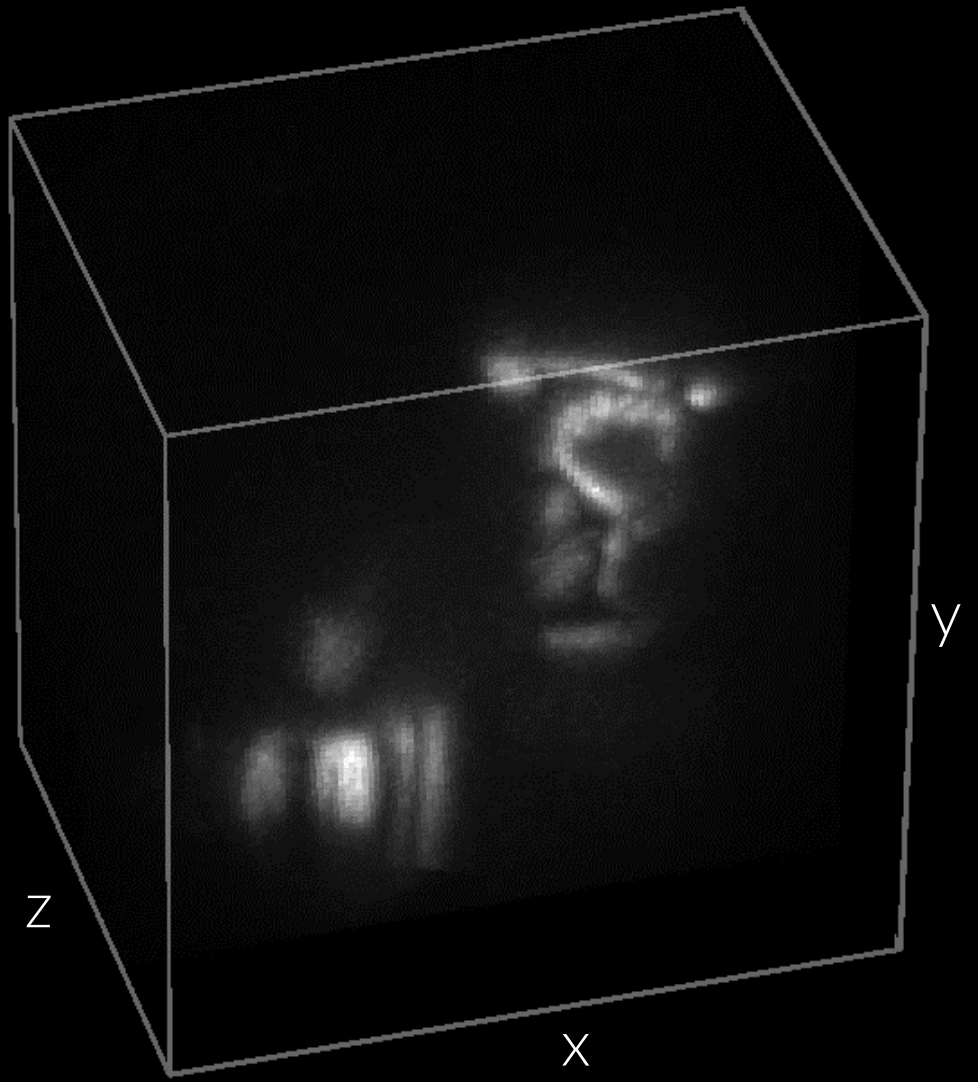
measurements



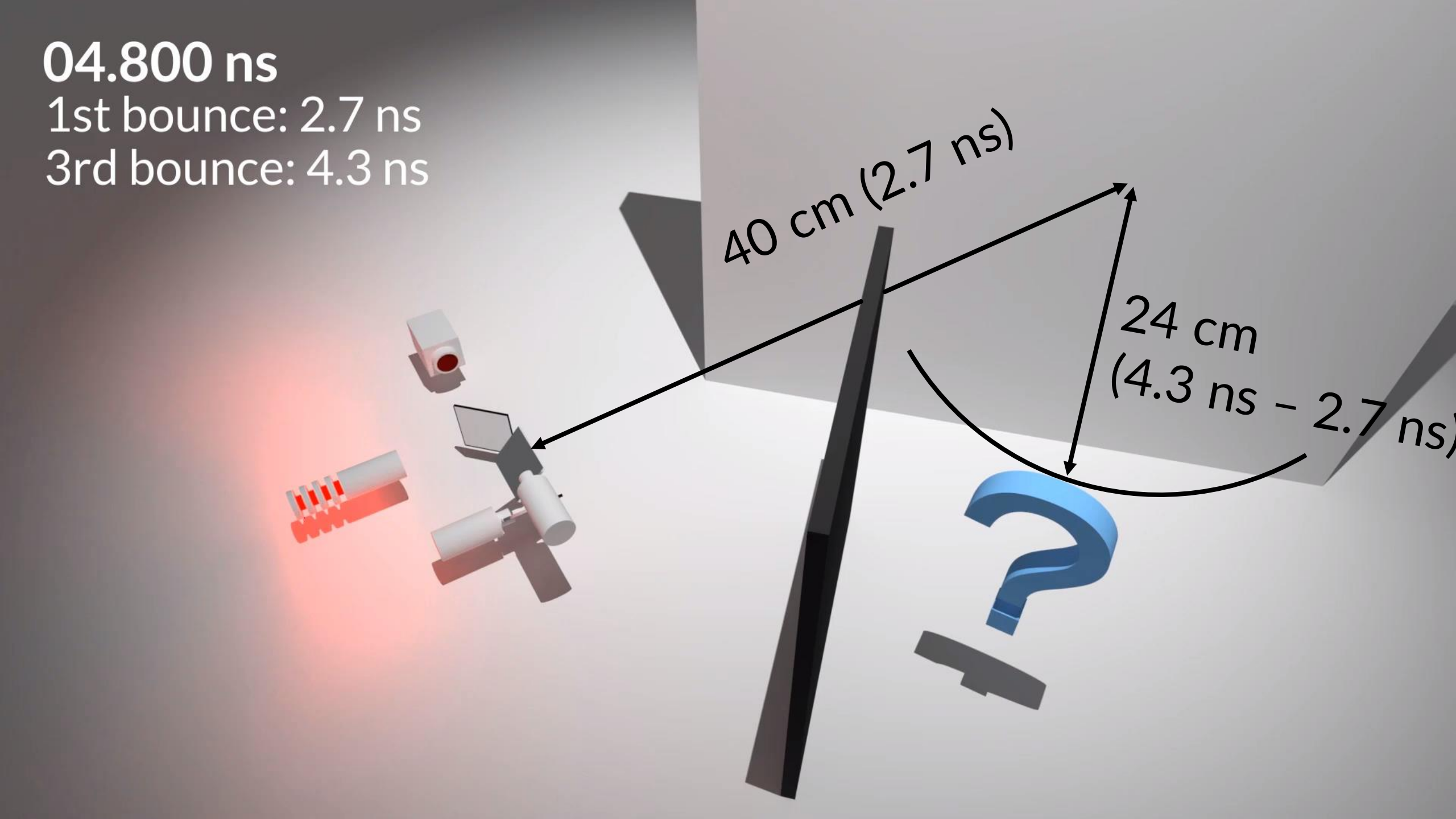
scene photo



reconstruction

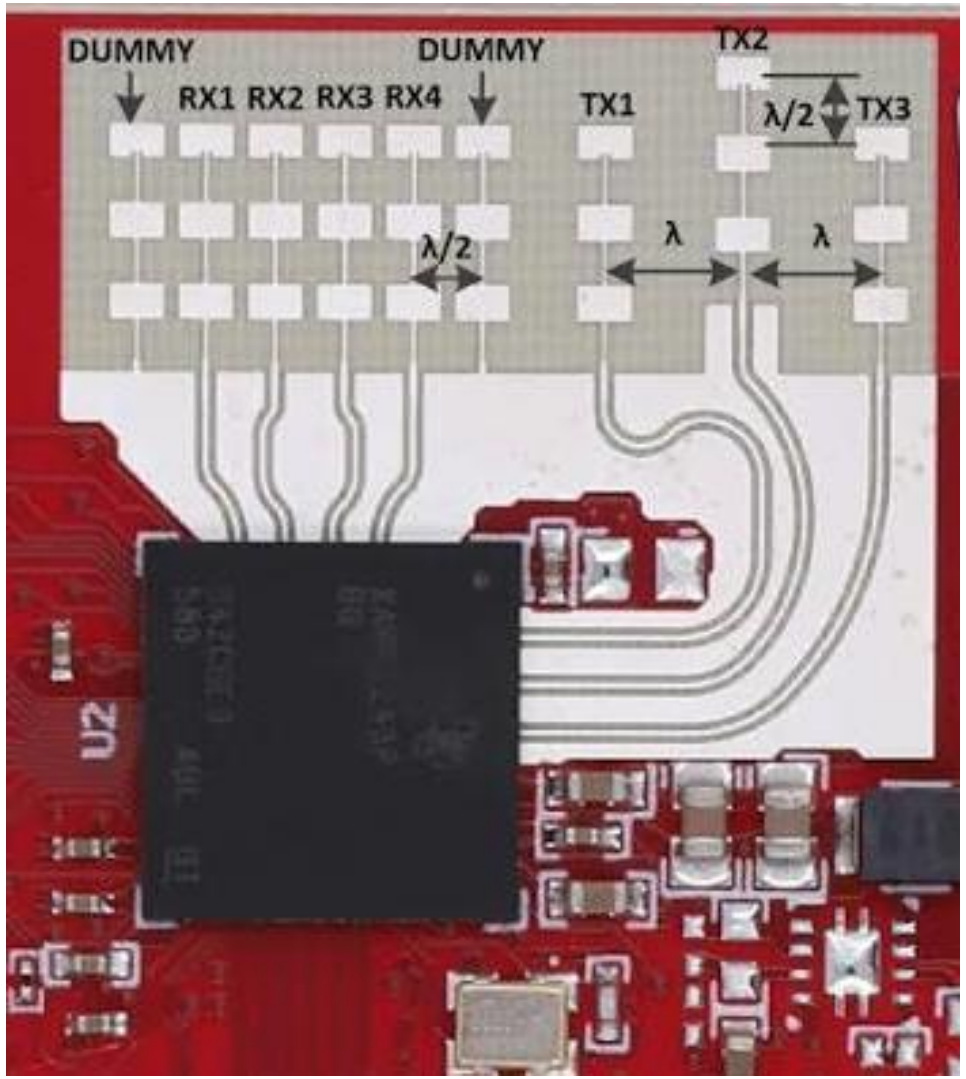


Dimensions: 2 m x 2 m x 1.5 m


04.800 ns
1st bounce: 2.7 ns
3rd bounce: 4.3 ns
40 cm (2.7 ns)
24 cm
(4.3 ns – 2.7 ns)

mmWave Review

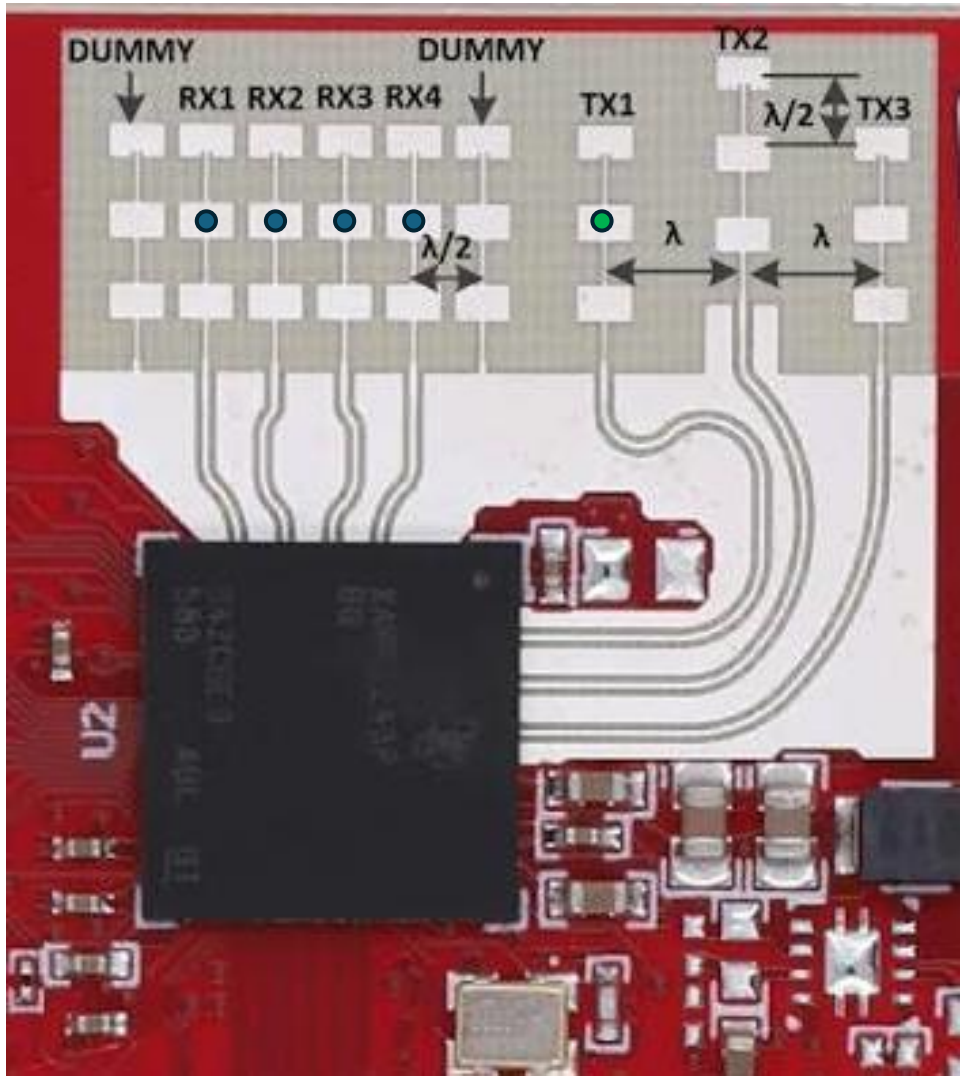
TI AWR2243 mmWave Sensor



- 4 GHz available bandwidth
- 3 transmitters + 4 receivers

Question: How do we increase the angular resolution of this sensor?

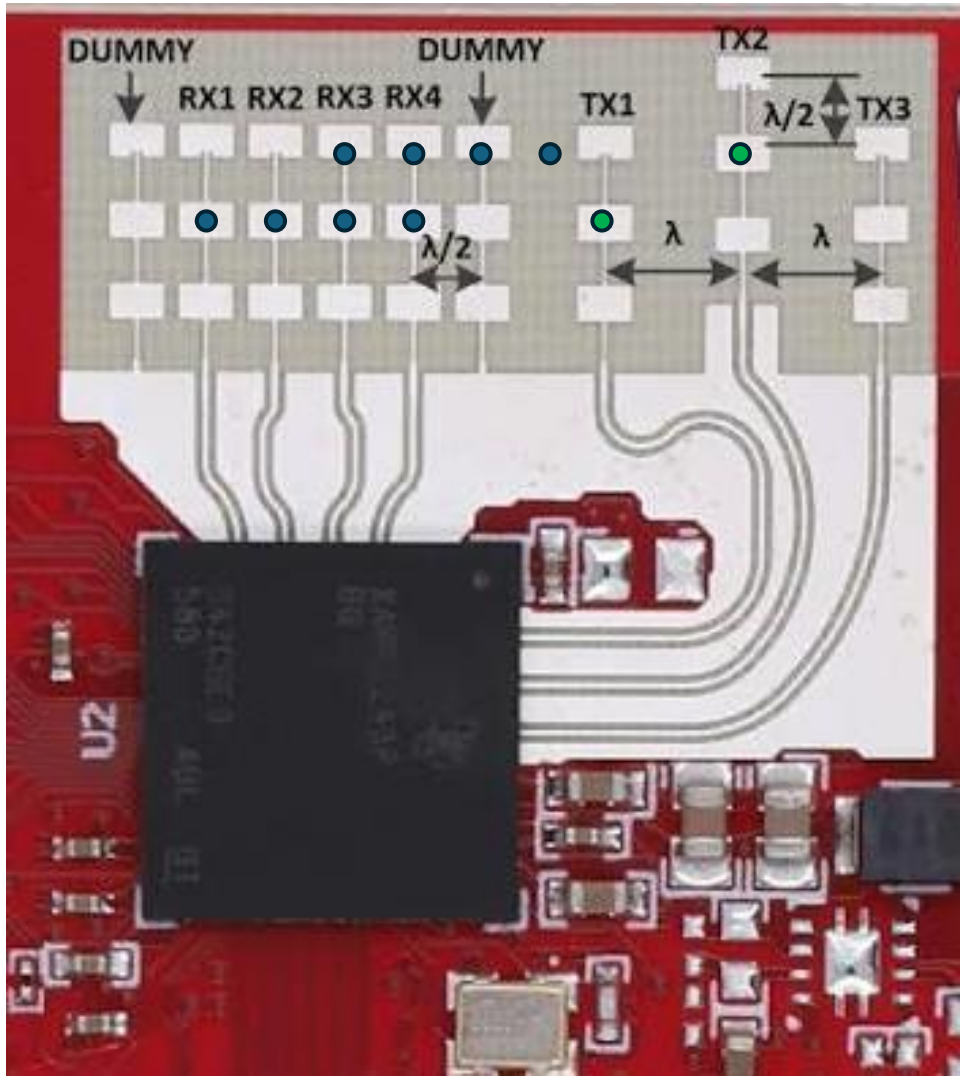
TI AWR2243 mmWave Sensor



- 4 GHz available bandwidth
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Question: How do we increase the angular resolution of this sensor?

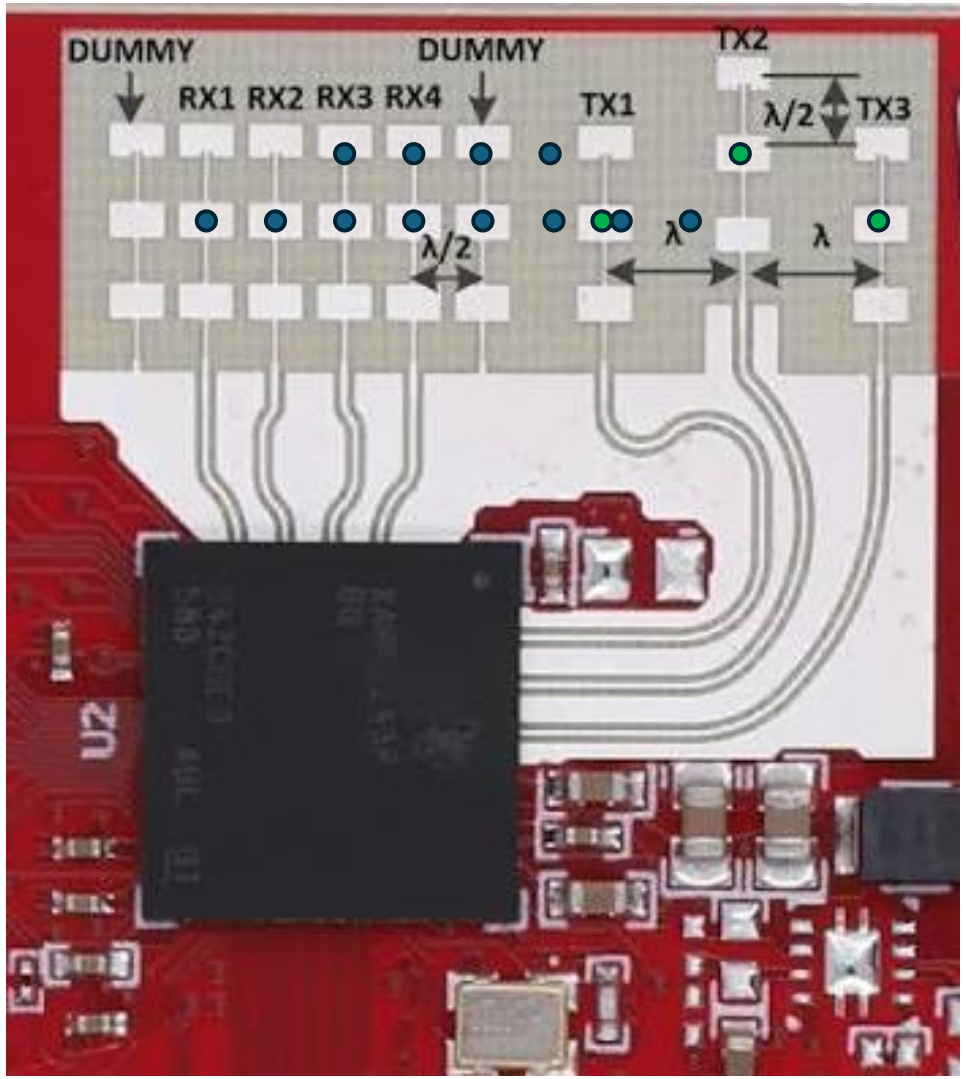
TI AWR2243 mmWave Sensor



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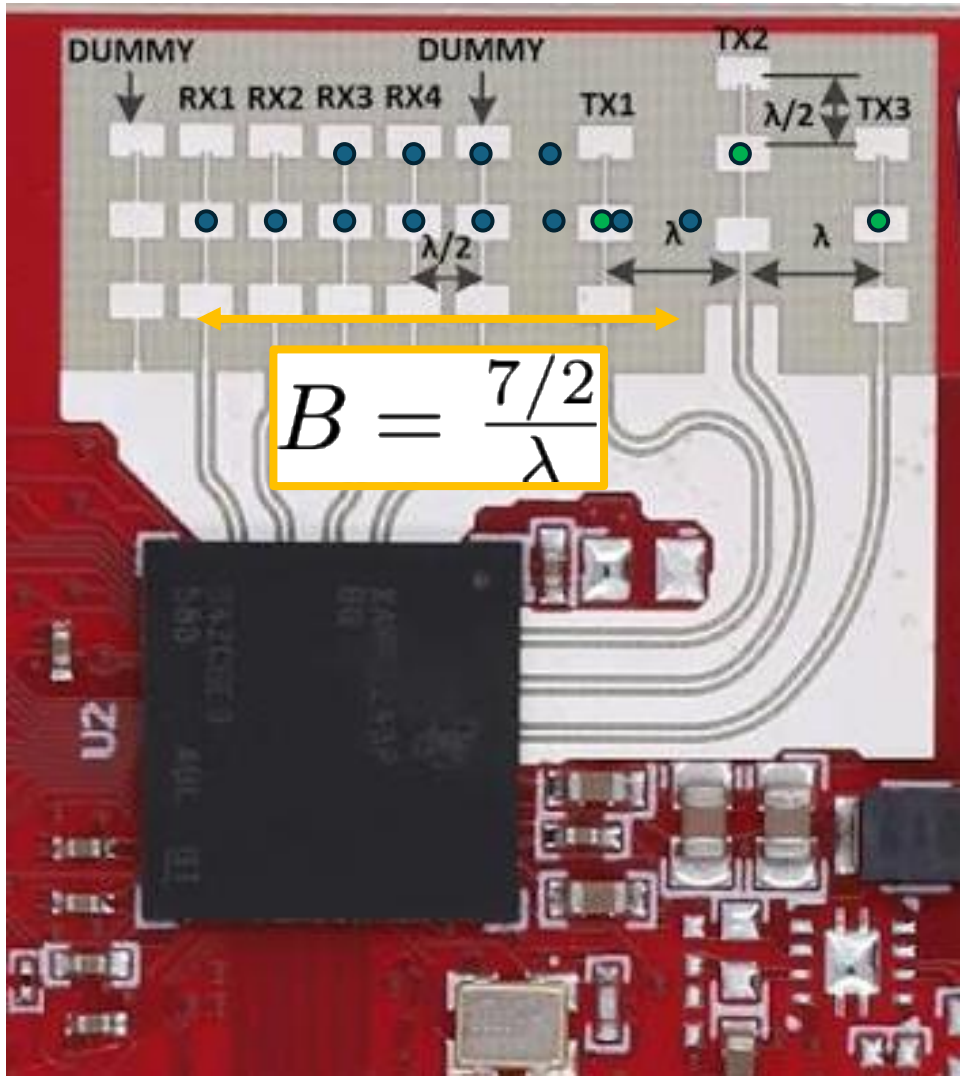
TI AWR2243 mmWave Sensor



- 4 GHz available bandwidth
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TI AWR2243 mmWave Sensor



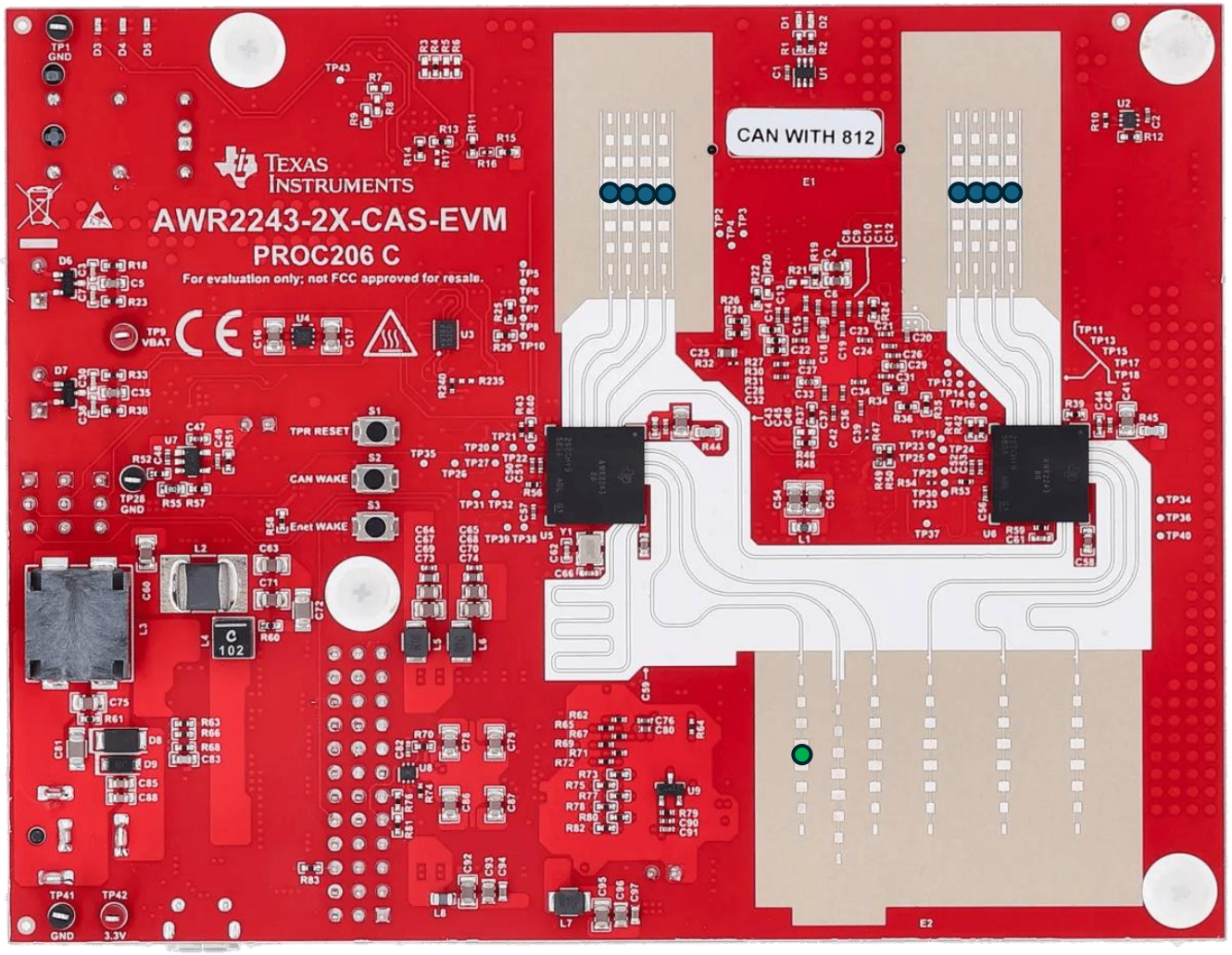
- 4 GHz available bandwidth
- 3 transmitters + 4 receivers

Question: How do we increase the angular resolution of this sensor?

- Angular resolution:

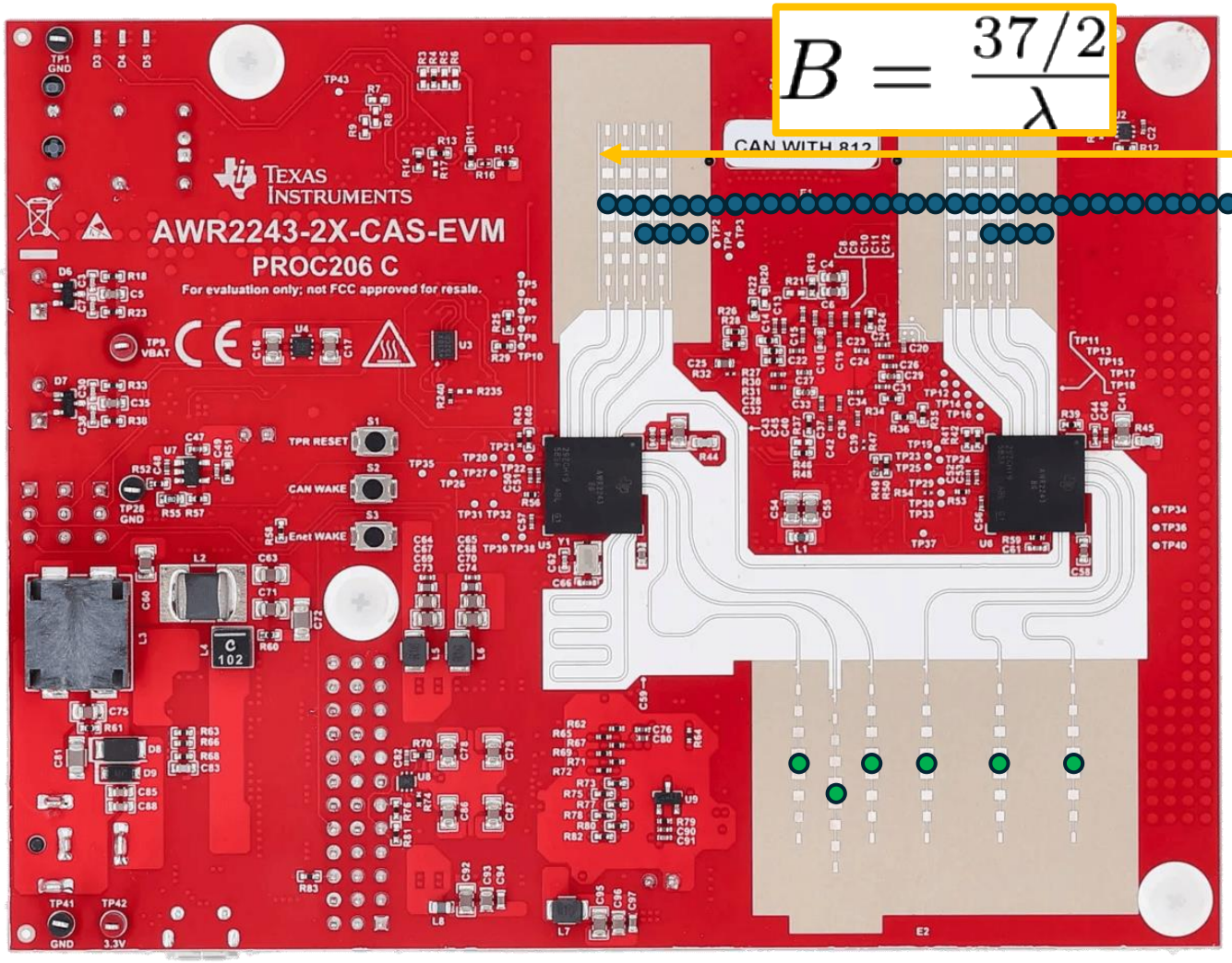
$$\theta \approx \frac{\lambda}{B}$$

TI AWR2243 Cascaded mmWave Sensor



- 4 GHz available bandwidth
- 2-chip cascade
  - 6 transmitters and 8 receivers

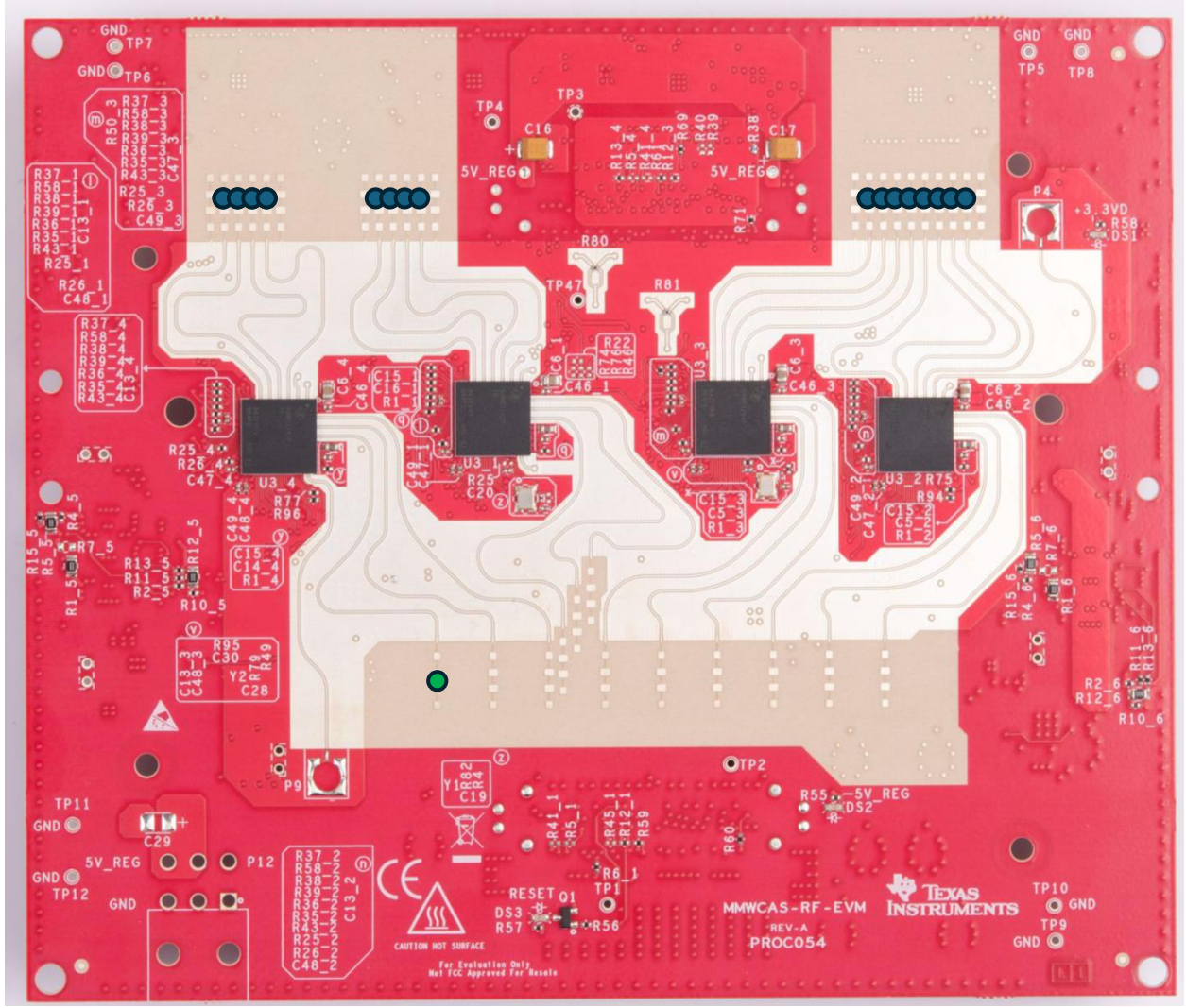
TI AWR2243 Cascaded mmWave Sensor



$$B = \frac{37/2}{\lambda}$$

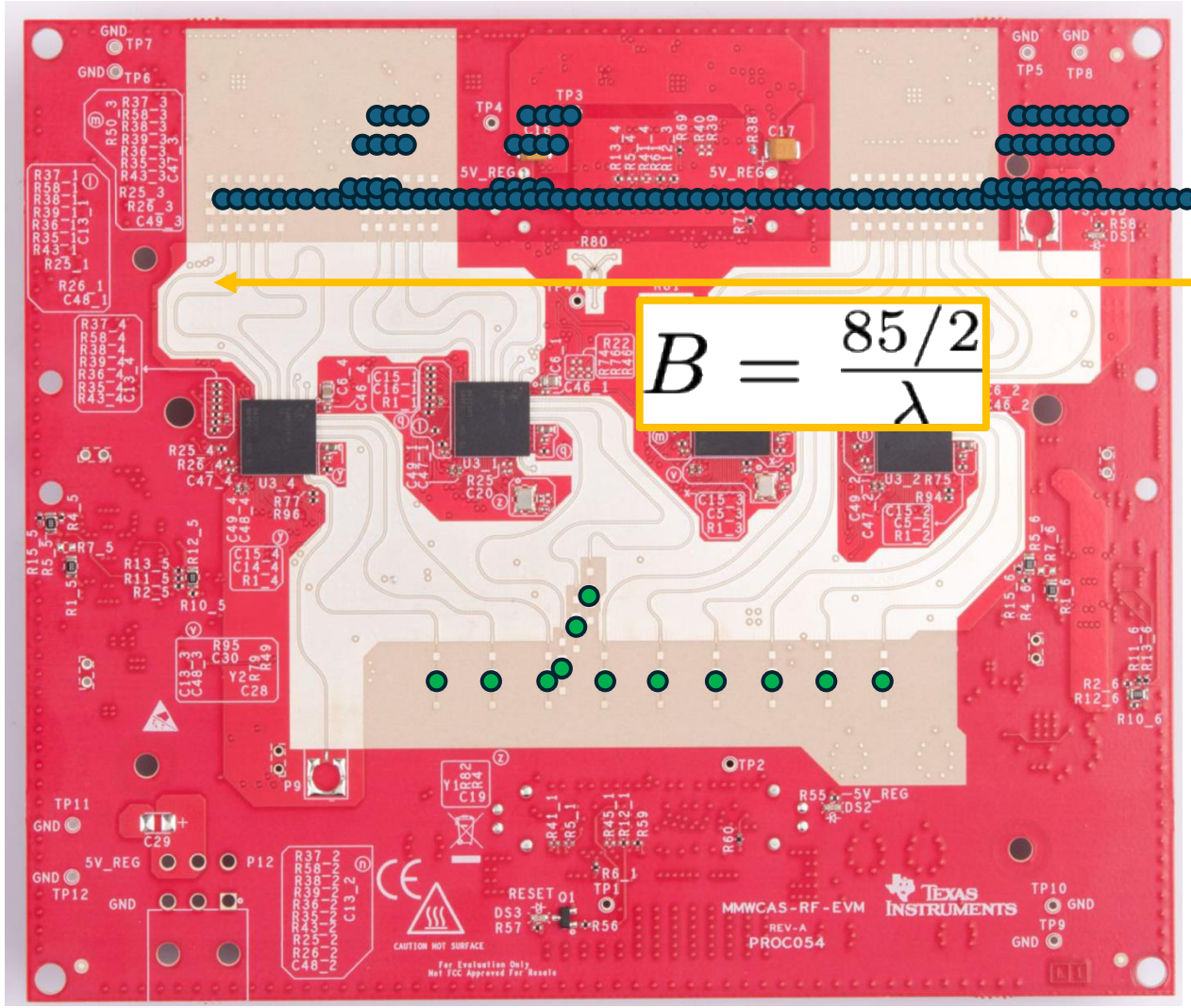
- 4 GHz available bandwidth
- 2-chip cascade
  - 6 transmitters and 8 receivers
  - About 37/7 = 5.28 higher angular resolution over 1-chip

TI AWR2243 Cascaded mmWave Sensor

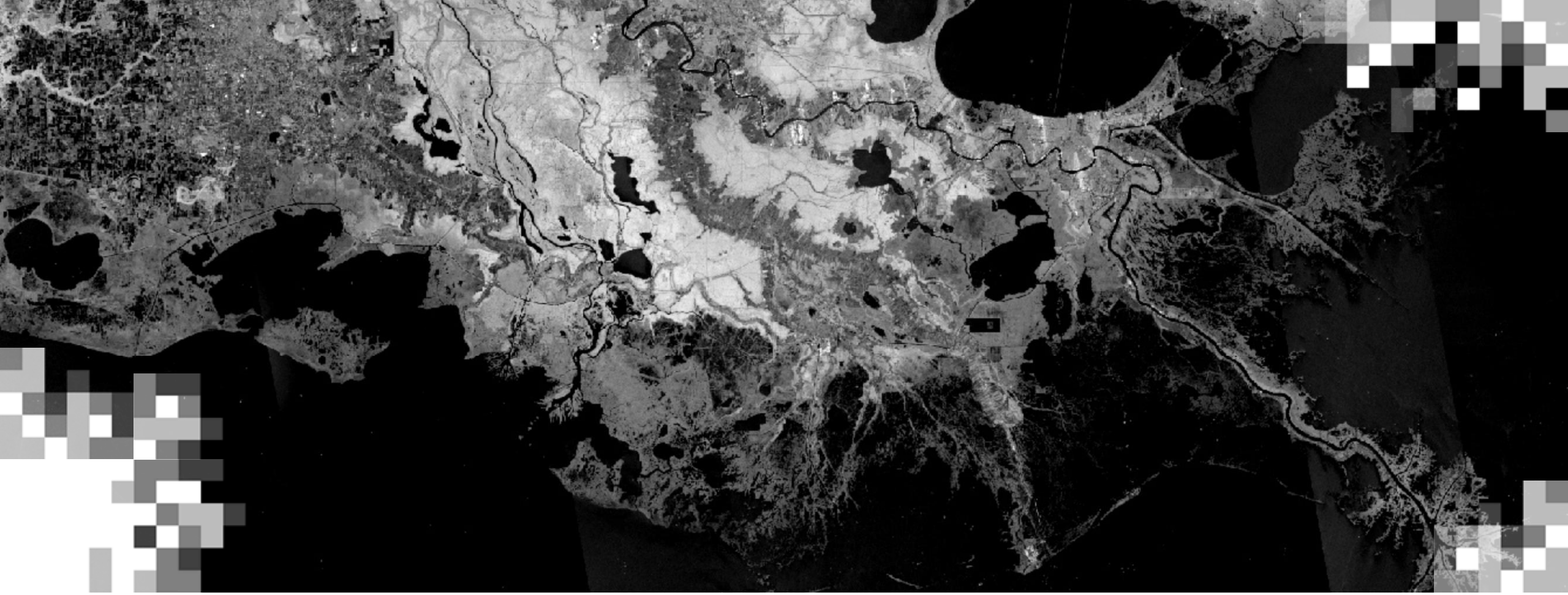


- 4 GHz available bandwidth
- 4-chip cascade
  - 12 transmitters and 16 receivers

TI AWR2243 Cascaded mmWave Sensor



- 4 GHz available bandwidth
- 4-chip cascade
  - 12 transmitters and 16 receivers
  - About 85/7 = 12.14 higher angular resolution over 1-chip



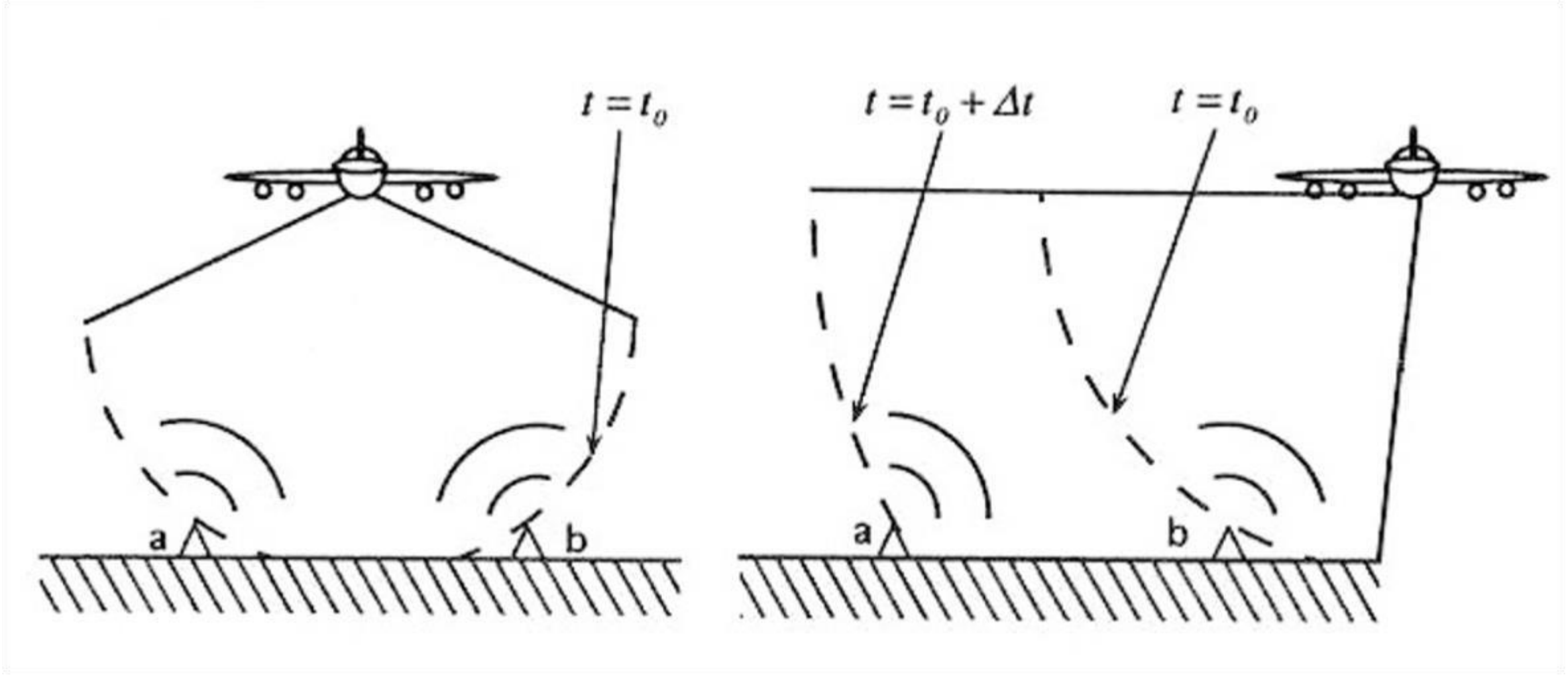
Side Looking Airborne Radar

Slide Credit

Erika Podest (NASA)


UNITED STATES OF AMERIC

Down-Looking vs. Side-Looking Radar



Side-Looking Radar

- Each pixel in the radar image represents a complex quantity of the energy that was reflected back to the satellite.
- The magnitude of each pixel represents the intensity of the reflected echo.

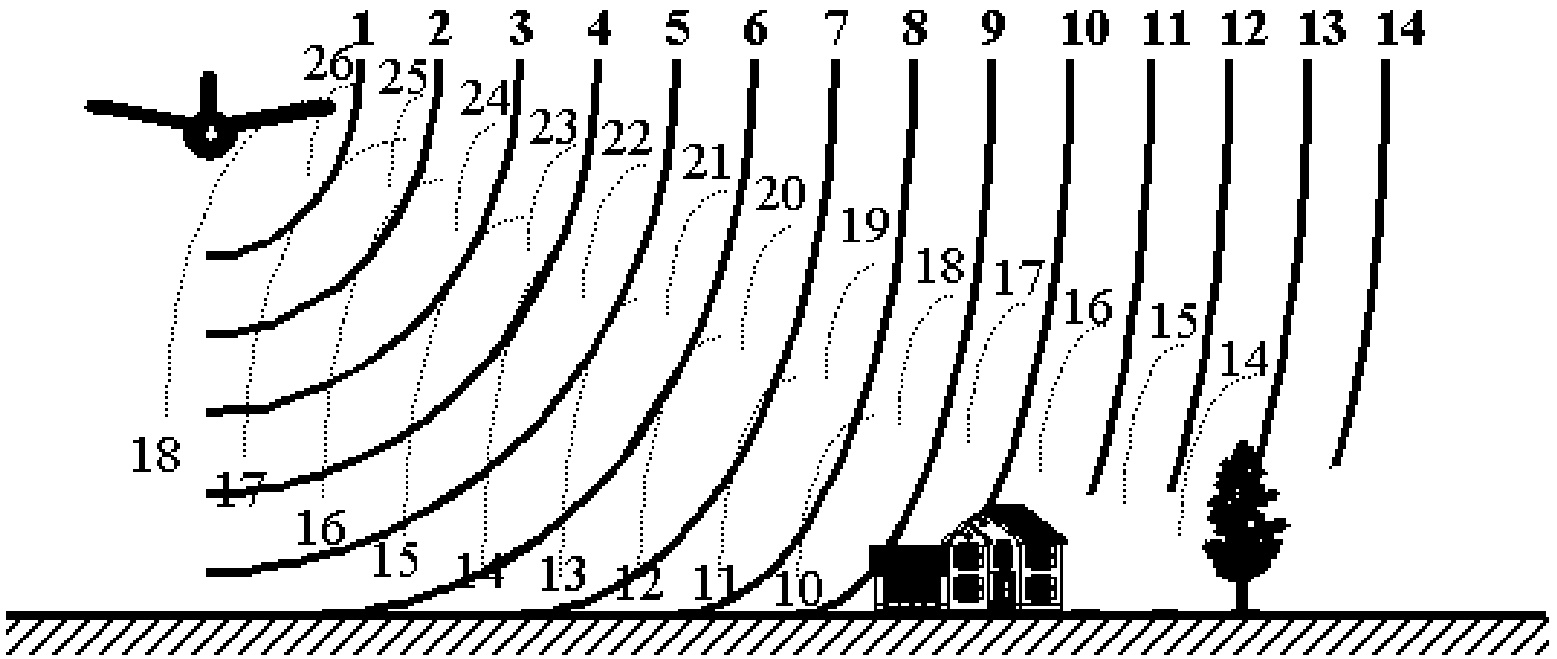


Image Credit: Lillesand and Kiefer, 1994

Radar Geometry

Slant Range Distance

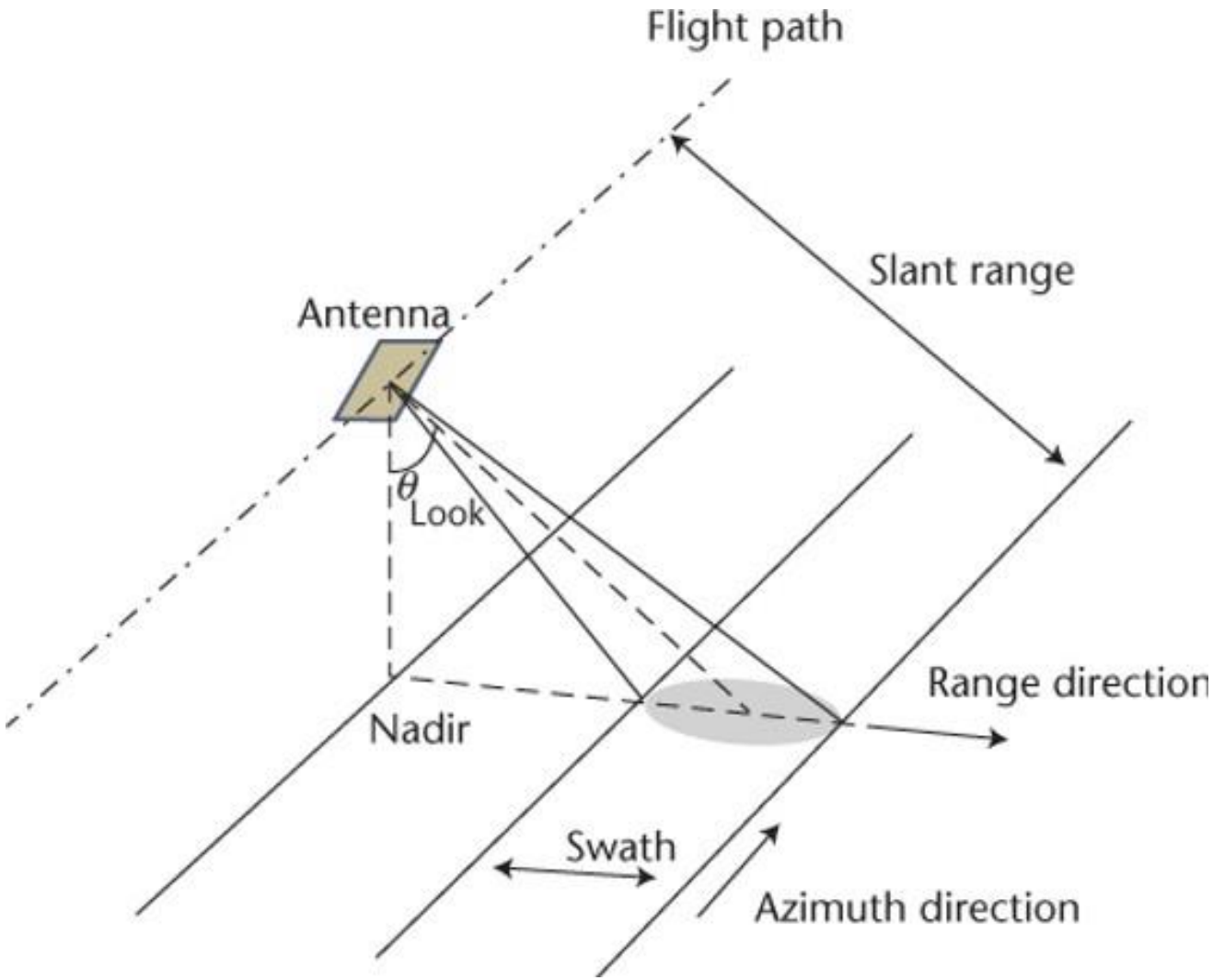
- The direct distance between the sensor and the target

Range Direction (Across Track)

- The direction perpendicular to the motion of the radar sensor

Azimuth Direction (Along Track)

- The direction parallel to the motion of the radar platform as it moves over the target area



Range Resolution

Range Resolution (Across Track)

- **Pulse Length:** The duration of the transmitted pulse.
- **Range resolution** depends on the length of the pulse (shorter pulses results in higher resolutions).
- If pulses are shortened, transmitted amplitude must be increased to maintain same total power in the pulse.
- Use range pulse compression by frequency modulation.

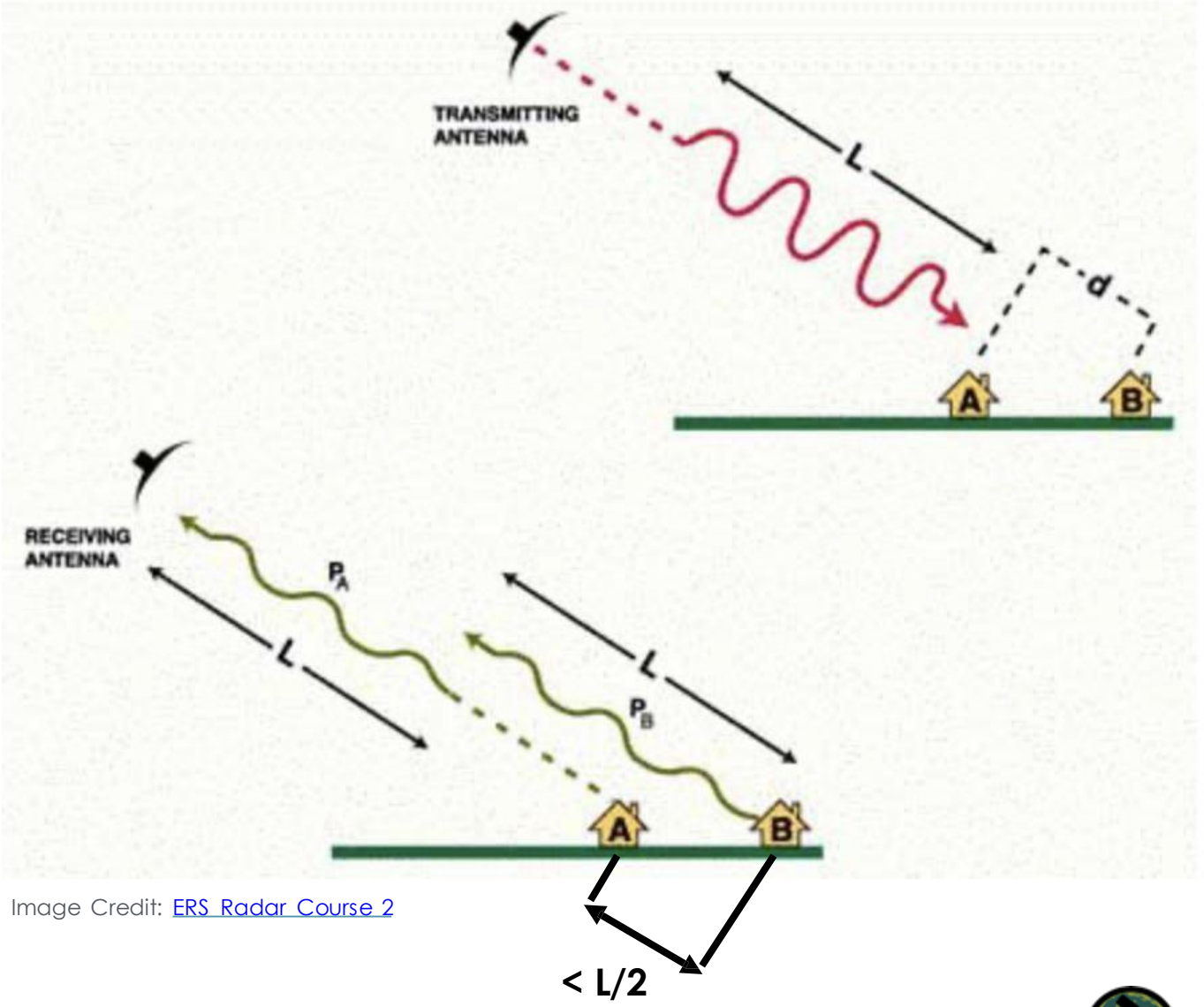


Image Credit: ERS Radar Course 2

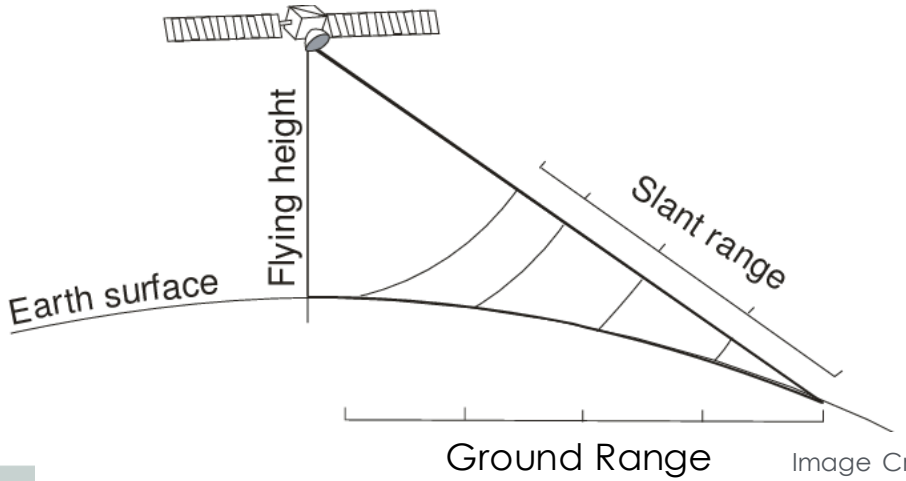
Slant Range Resolution vs. Ground Range Resolution

Ground Range

- Distance between the satellite ground track and the target

Incidence Angle

- Angle between line of sight of the radar and the vertical to the terrain



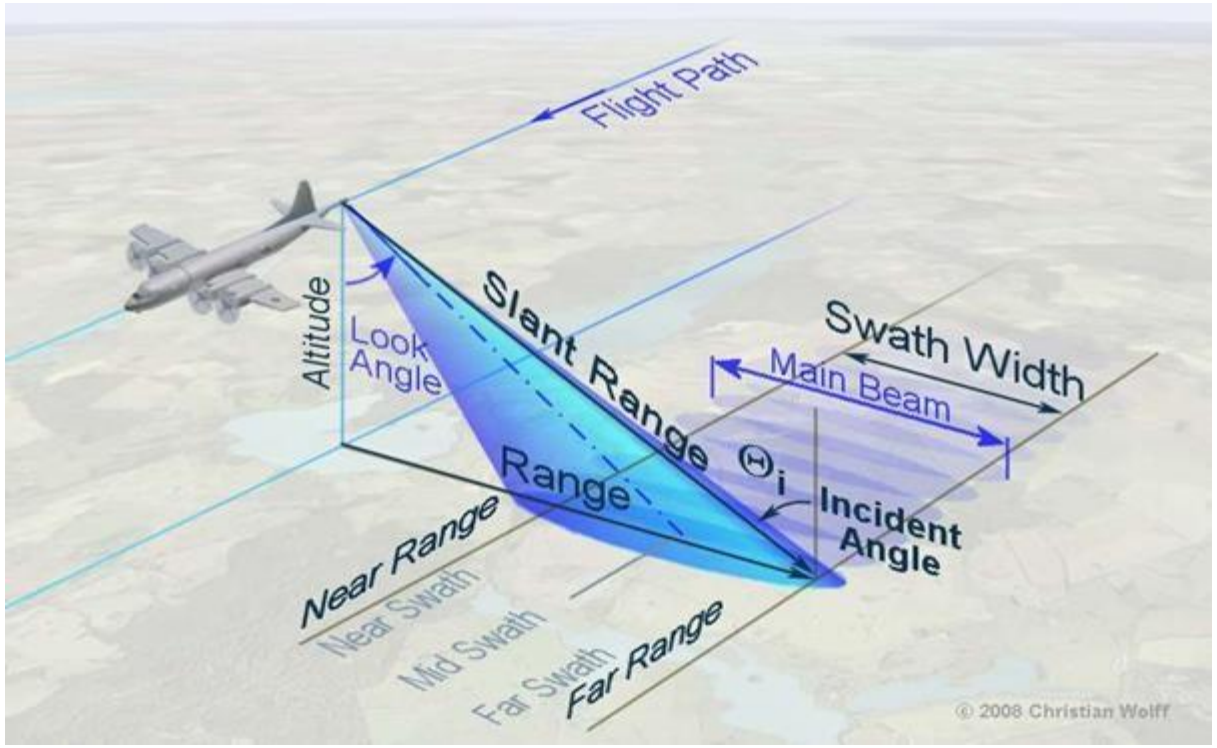


Image Credit: Silva et al., 2006

Azimuth Resolution

Azimuth Resolution (Along Track)

- Resolution of an imaging radar system in the azimuth direction is determined by the angular beamwidth and the slant-range.
- The beamwidth of the antenna is directly proportional to the wavelength of the transmitted pulse and inversely proportional to the length of the antenna (or aperture).

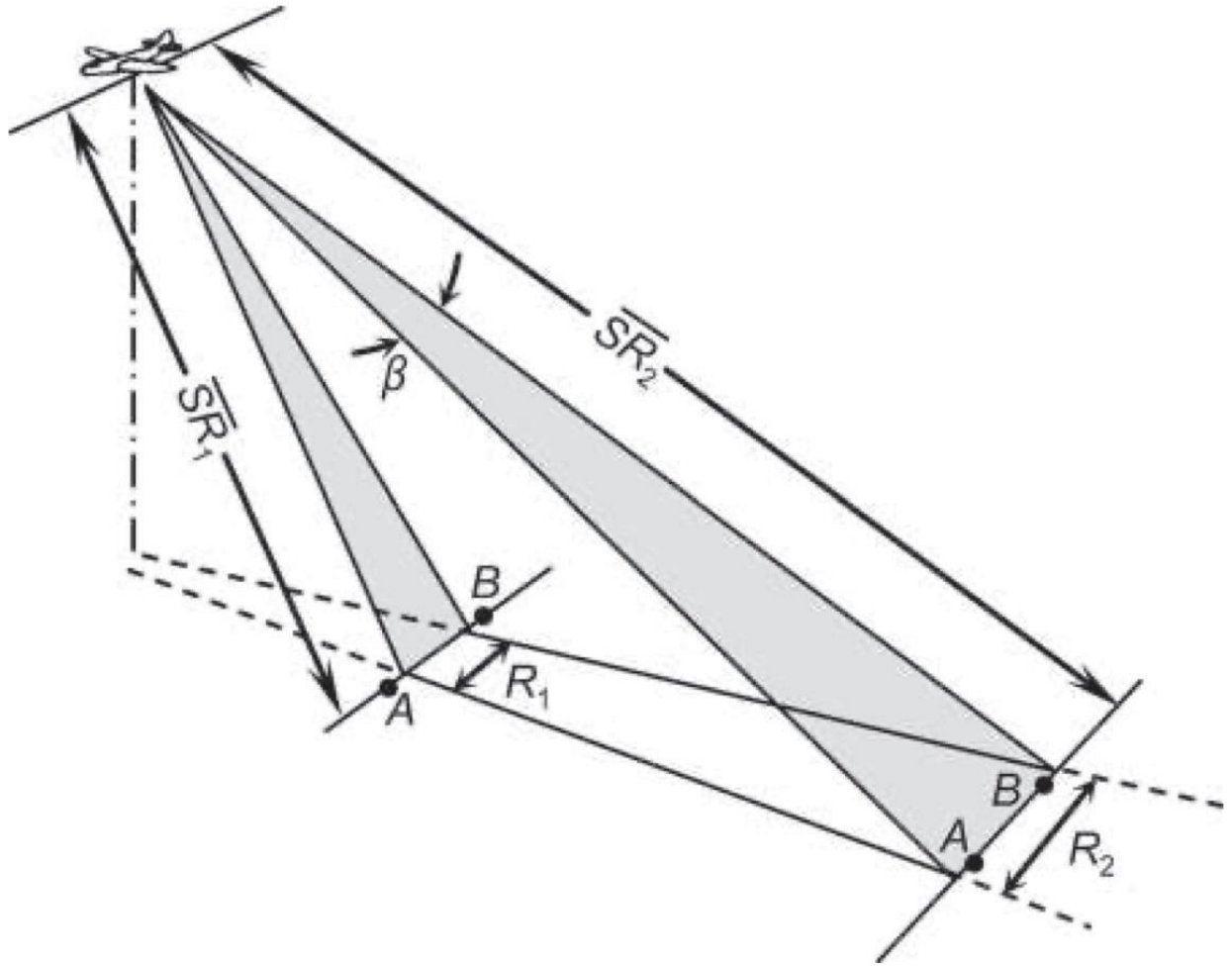


Image Credit: Lillesand and Kiefer, 7th Edition

Synthetic Aperture Radar

Azimuth Resolution (Along Track)

- It is difficult to have long antennas in space, but one can be synthesized by using the movement of the satellite to simulate a very long antenna.

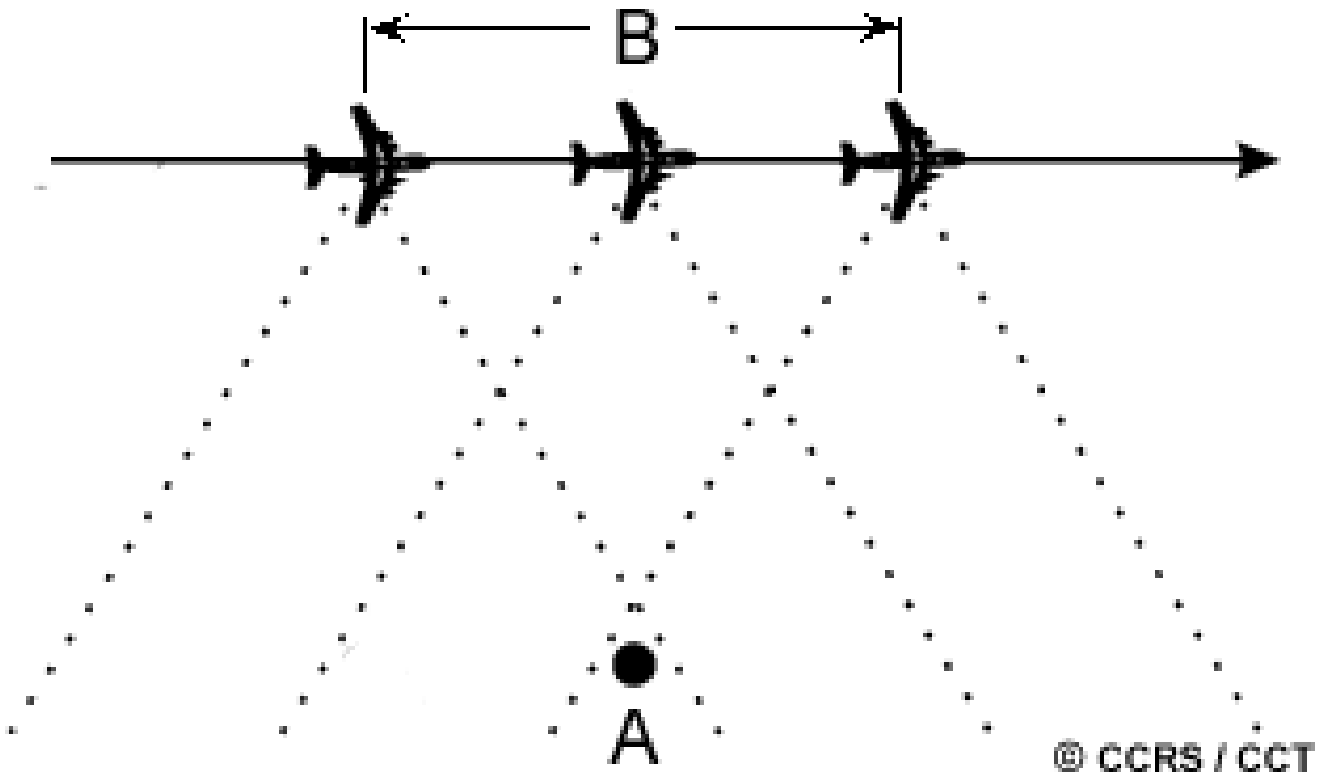
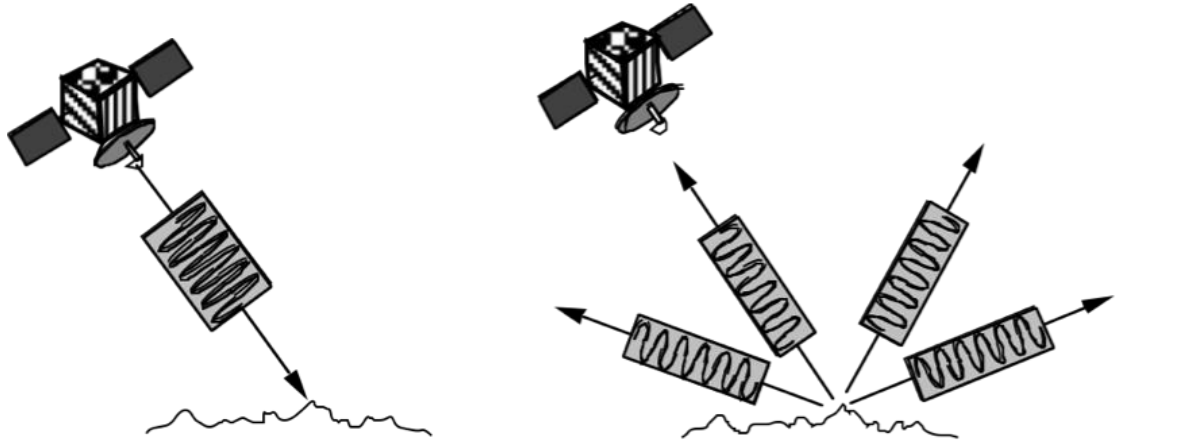


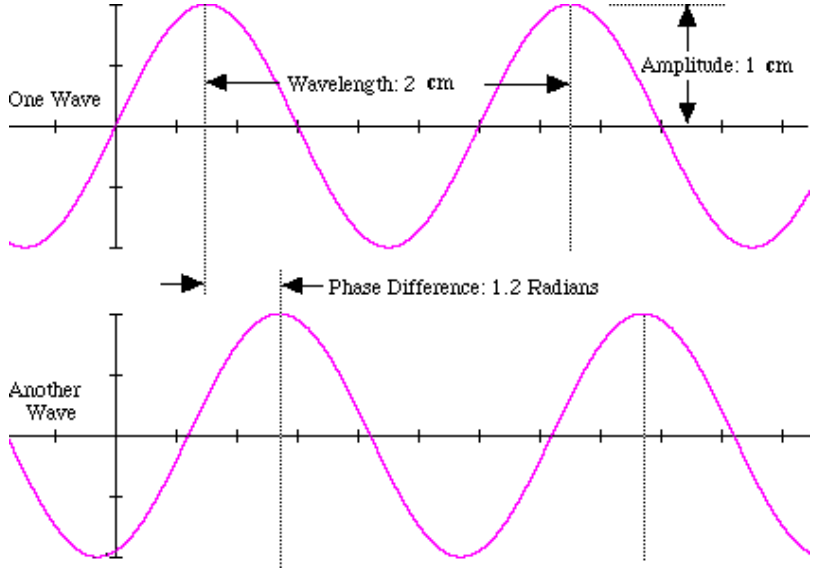
Image Credit: Lillesand and Kiefer, 7th Edition

Review of Radar Image Formation

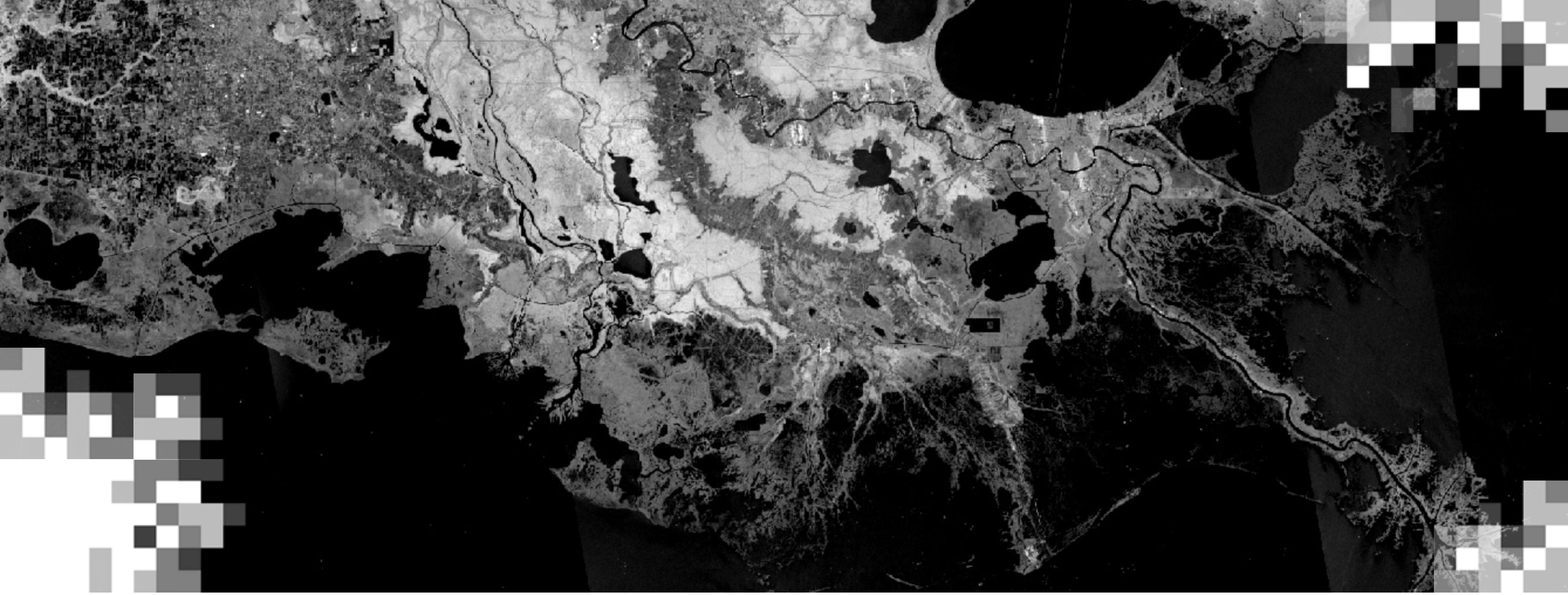
1. Radar can measure amplitude (the strength of the reflected echo) and phase (the position of a point in time on a waveform cycle).
2. Radar can only measure the part of the echo reflected back towards the antenna (backscatter).
3. Radar pulses travel at the speed of light.
4. The strength of the reflected echo is the backscattering coefficient (sigma naught) and is expressed in decibels (dB).







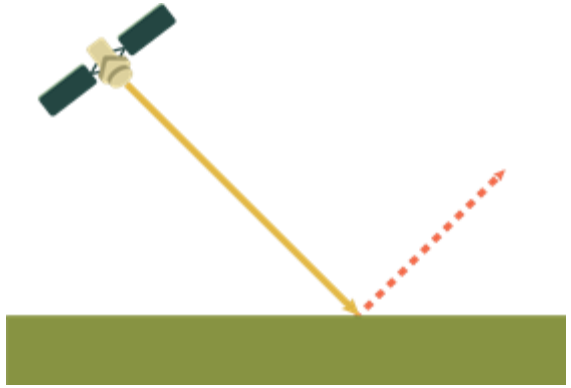
Source: ESA-ASAR Handbook



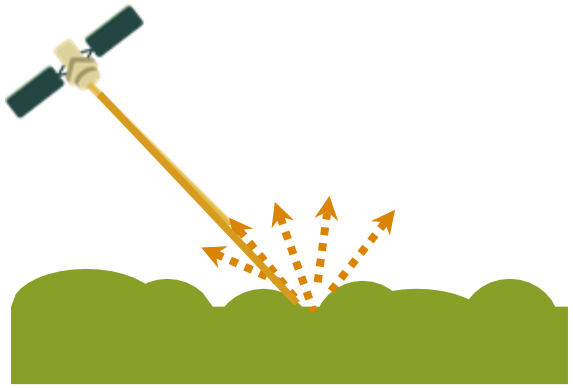
Radar Backscatter Mechanisms

Radar Signal Interaction

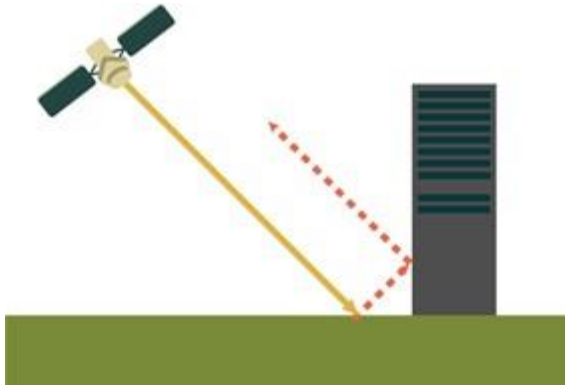
- The scale of the surface relative to the wavelength determines how rough or smooth it appears, as well as how bright or dark it will be in the image.
- Backscattering Mechanisms:



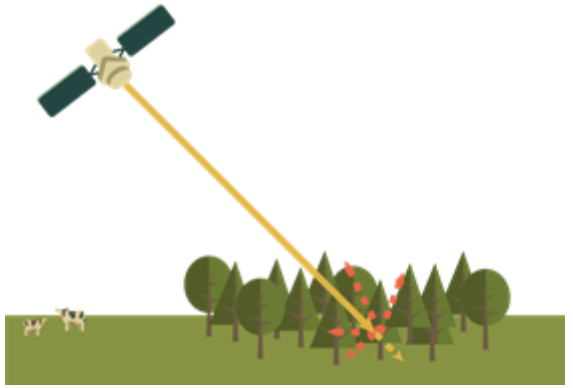
Smooth Surface
(specular Reflector)



Rough Surface

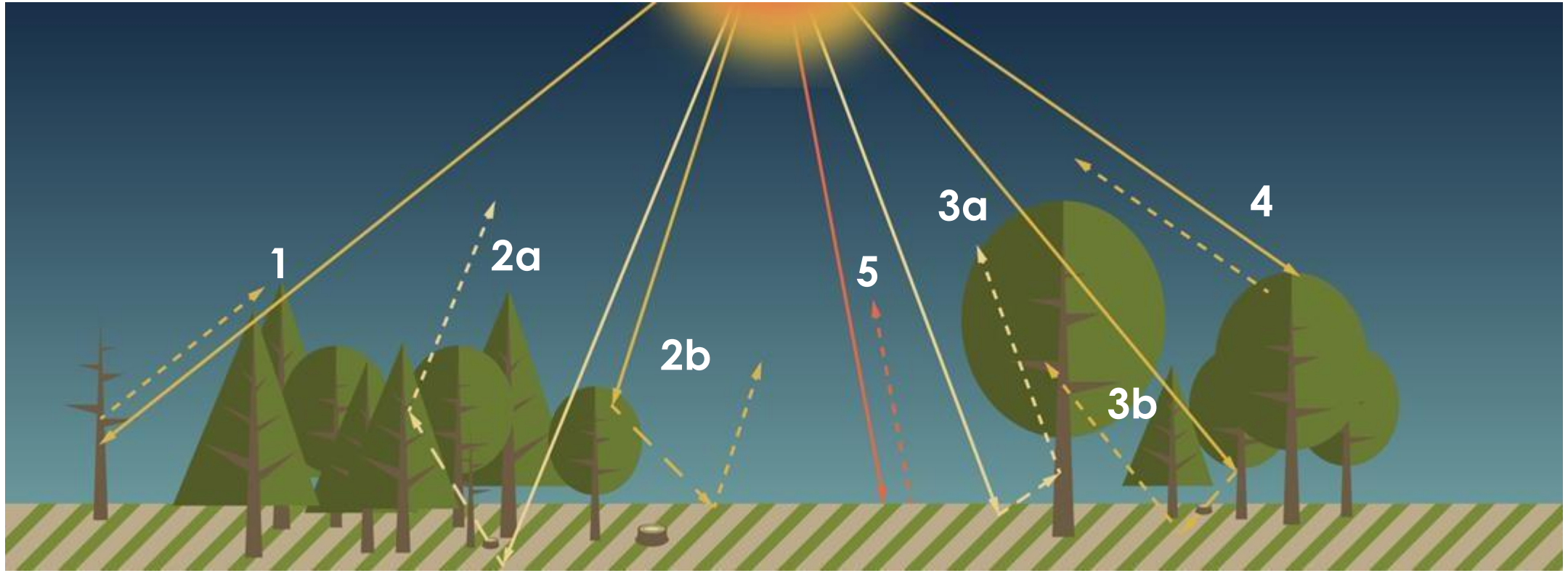


Double Bounce



Volumetric

Volumetric Scattering: Radar Backscatter in Forests

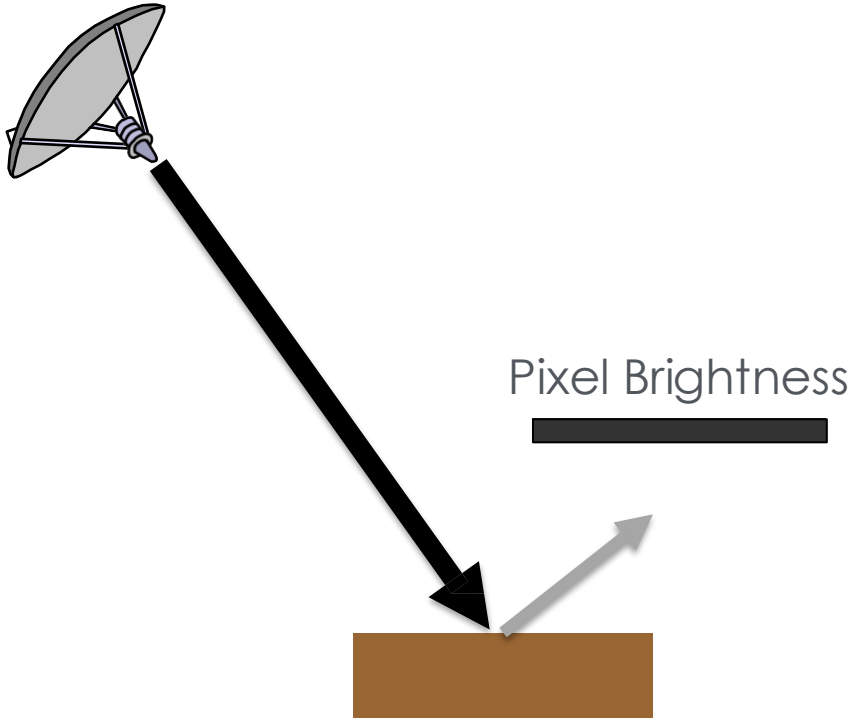


Dominant Backscattering Sources in Forests:

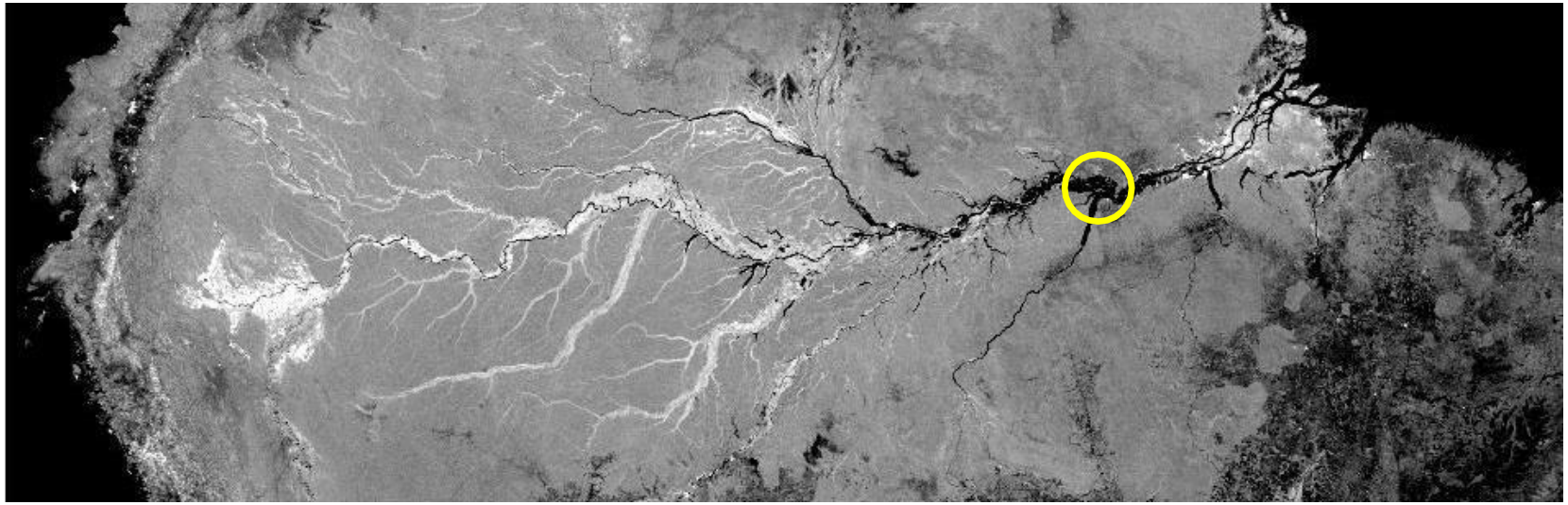
(1) Direct Scattering from the Tree Trunks
(2a) Ground-Crown Scattering
(2b) Crown-Ground Scattering
(3a) Ground-Trunk Scattering
(3b) Trunk-Ground Scattering
(4) Crown Volume Scattering
(5) Direct Scattering from the Soil Surface

Smooth Surface Reflection

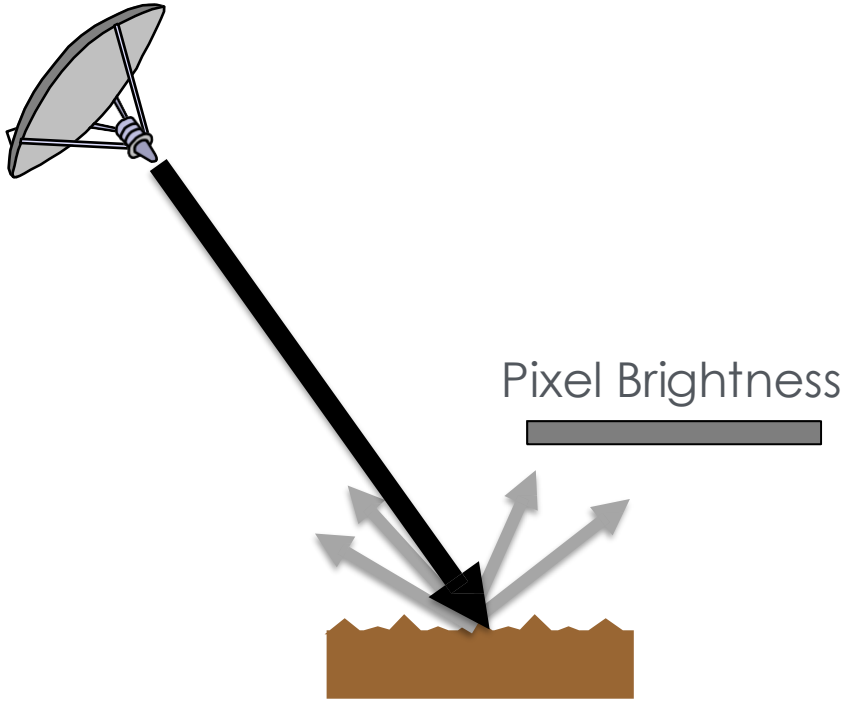
Smooth Surface Reflection (Specular Reflection)



SMAP Radar Mosaic of the Amazon Basin
April 2015 (L-band, HH, 3 km)

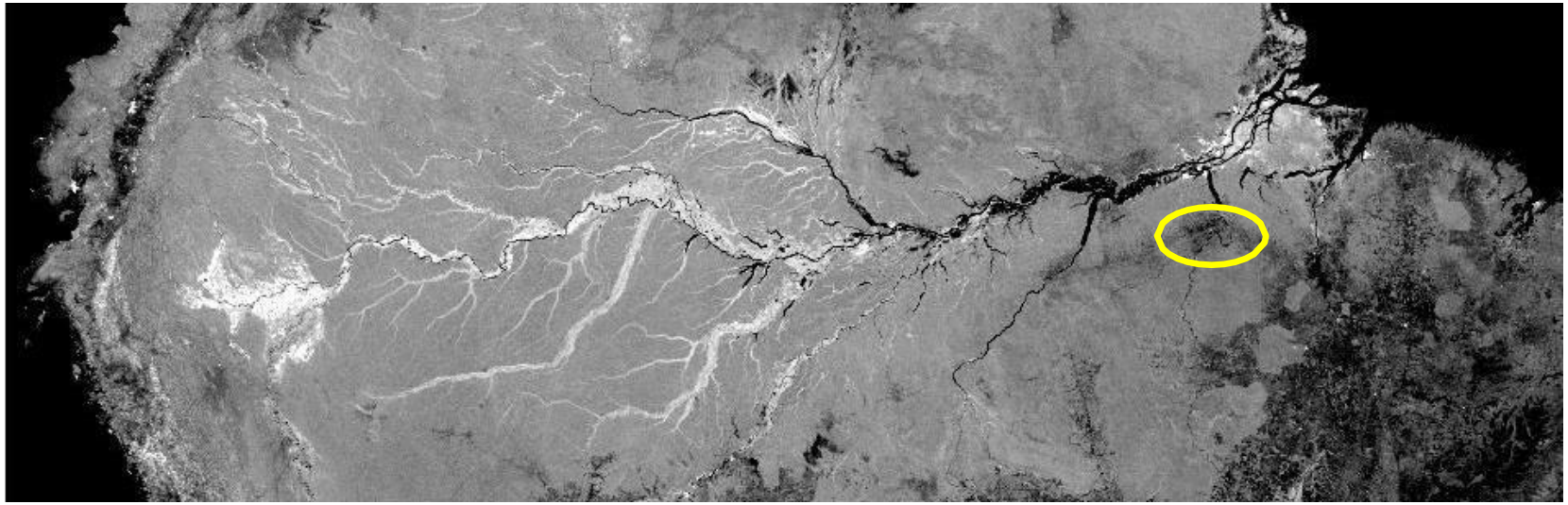


Rough Surface Reflection

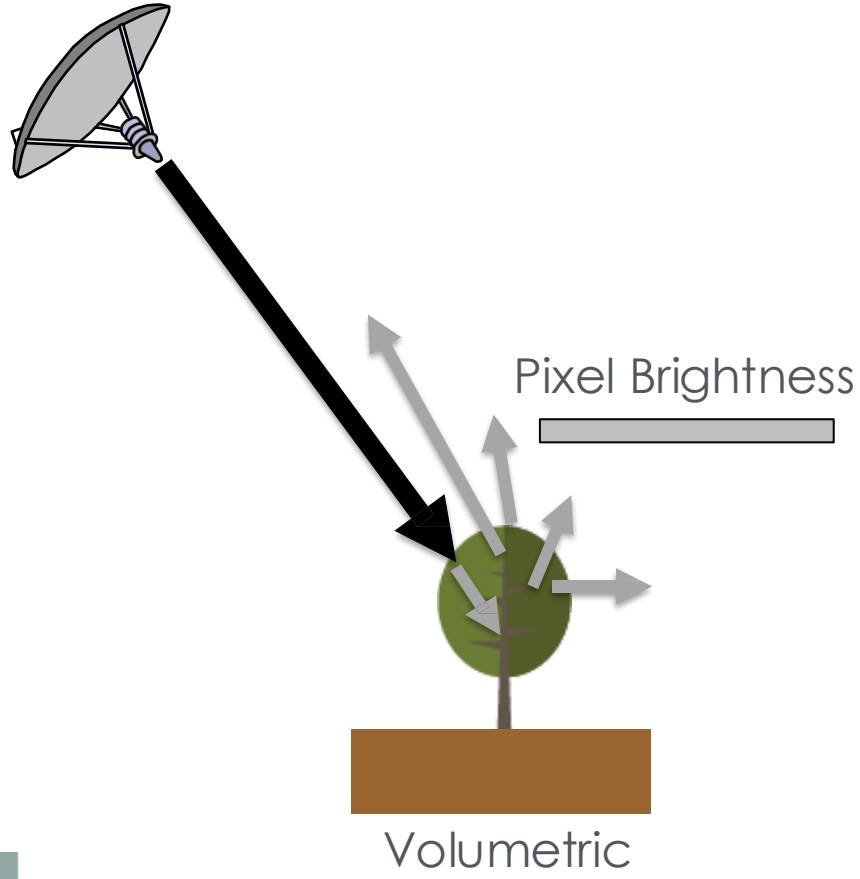


Rough, Bare Surface
(Deforested Areas, Tilled Agricultural Fields)

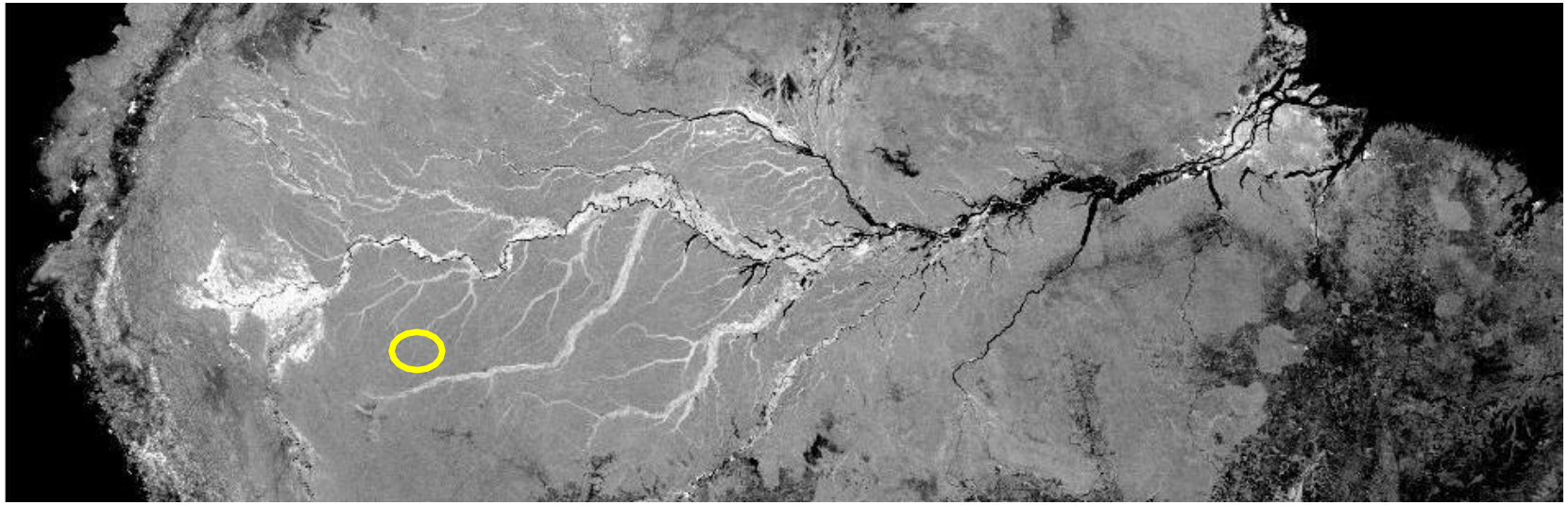
SMAP Radar Mosaic of the Amazon Basin
April 2015 (L-band, HH, 3 km)



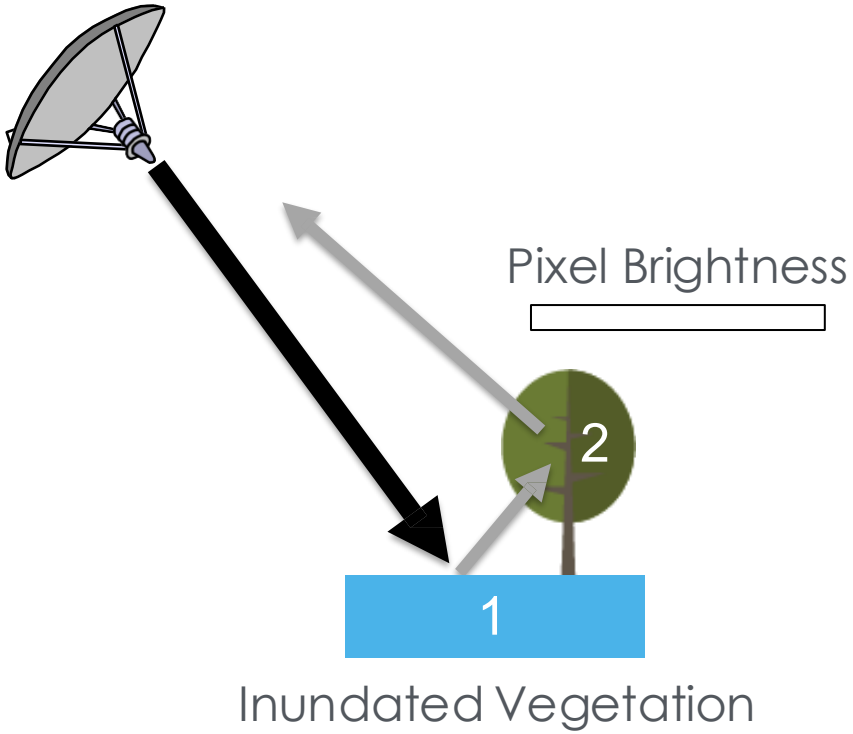
Volume Scattering by Vegetation



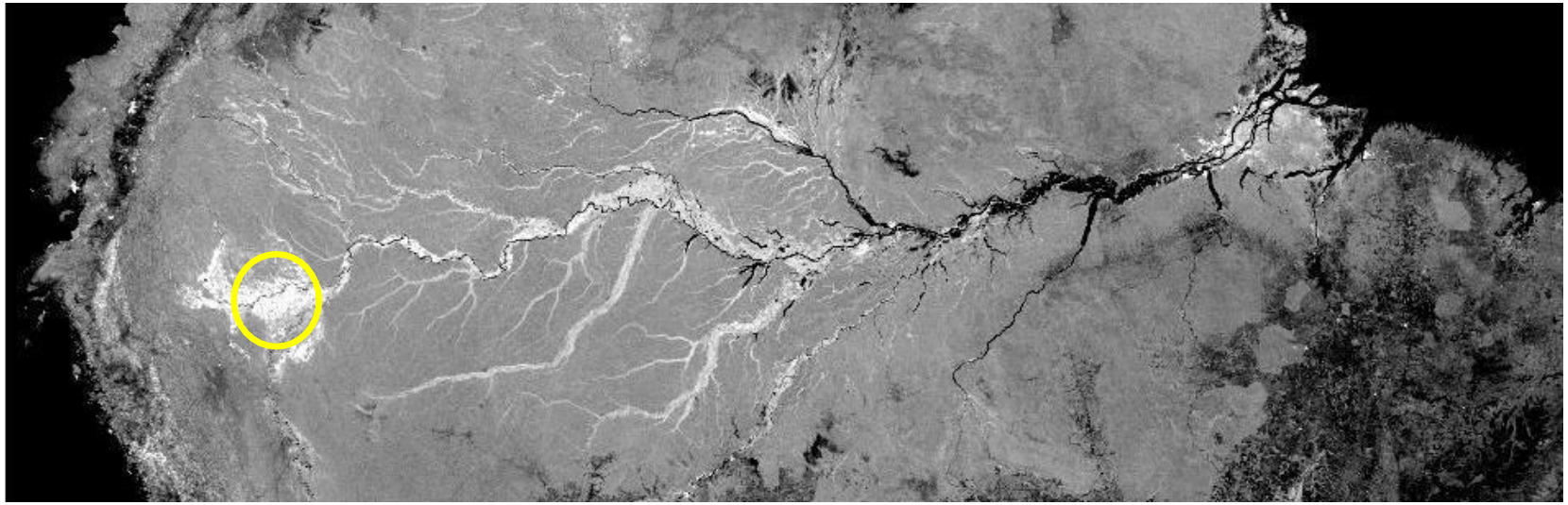
SMAP Radar Mosaic of the Amazon Basin
April 2015 (L-band, HH, 3 km)

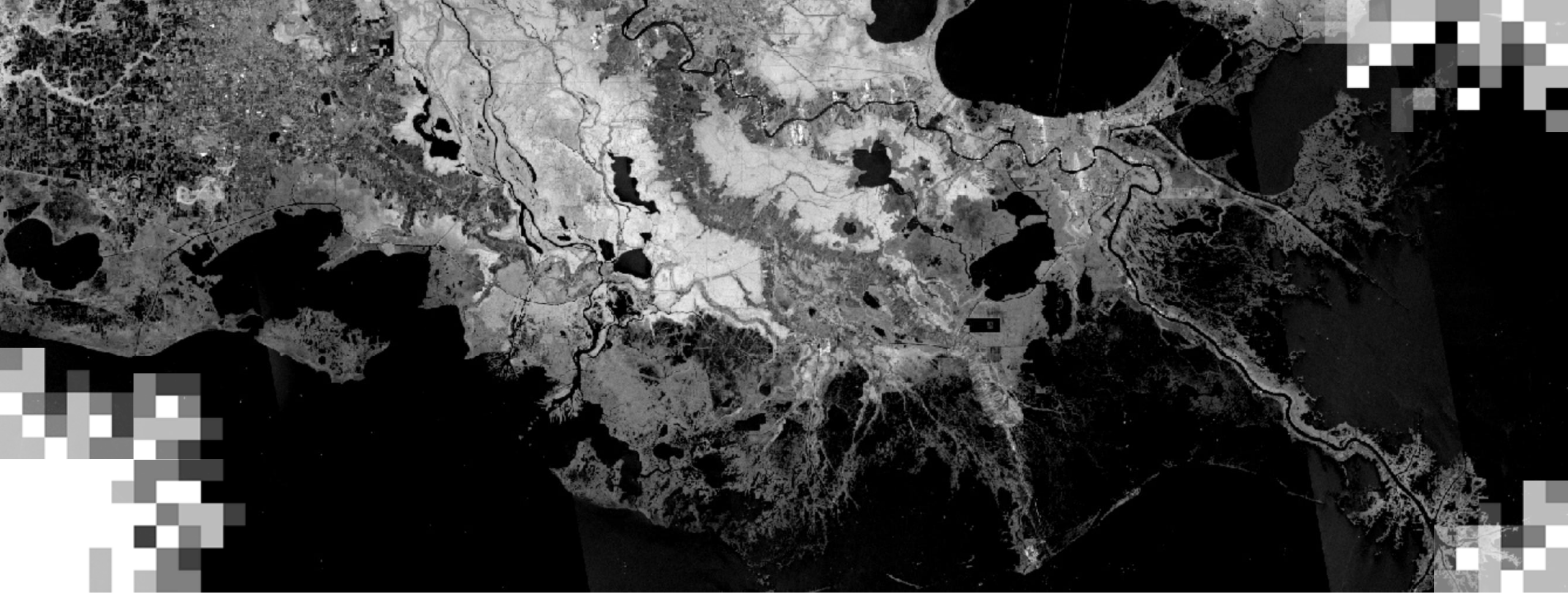


Double Bounce



SMAP Radar Mosaic of the Amazon Basin
April 2015 (L-band, HH, 3 km)





Radar Parameters that Influence the Signal

Radar Parameters to Consider for a Study

- Wavelength
- Polarization
- Incidence Angle

Radar Parameters: Wavelength

$$\lambda = \frac{c}{v}$$

c = speed of light (3×10^8 m/s)

λ = wavelength (m)

v = frequency (cycles per second, Hz)

Higher Frequency



Shorter Wavelength

Lower Frequency



Longer Wavelength

Band Designation*	Wavelength (λ), cm	Frequency (v), GH_z (10^9 cycles·sec^{-1})
Ka (0.86 cm)	0.8 – 1.1	40.0 – 26.5
K	1.1 – 1.7	26.5 – 18.0
Ku	1.7 – 2.4	18.0 – 12.5
X (3.0 cm, 3.2 cm)	2.4 – 3.8	12.5 – 8.0
C (6.0)	3.8 – 7.5	8.0 – 4.0
S	7.5 – 15.0	4.0 – 2.0
L (23.5 cm, 25 cm)	15.0 – 30.0	2.0 – 1.0
P (68 cm)	30.0 – 100.0	1.0 – 0.3

* Wavelengths most frequently used in SAR are in parentheses.

Signal Penetration as a Function of Wavelength

- Signal penetration is the **primary factor** in wavelength selection.
- Generally, the longer the wavelength, the greater the penetration into the target.

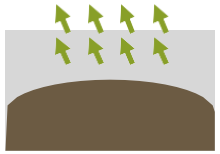
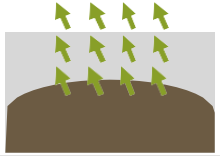
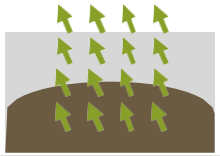
Vegetation			
Dry Alluvium			
	X-band 3 cm	C-band 5 cm	L-band 23 cm

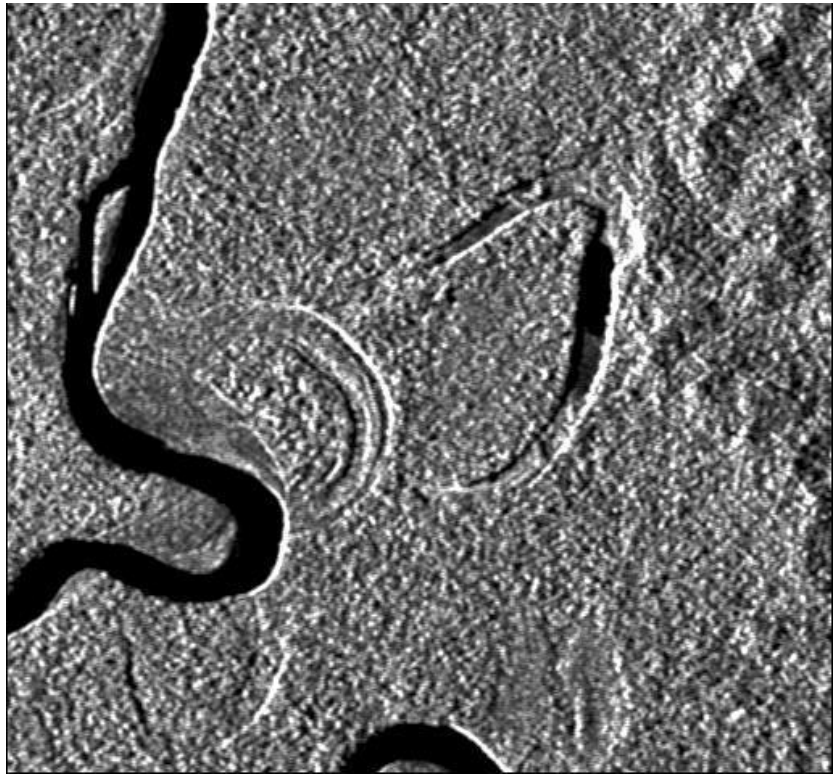
Image Based On ESA Radar Course 2

Frequency Band	Application Example
VHF	foliage & ground penetration, biomass
Frequency Band	Application Example
VHF	foliage & ground penetration, biomass
P-Band	biomass, soil moisture, penetration
L-Band	agriculture, forestry, soil moisture
C-Band	ocean, agriculture
X-band	agriculture, ocean, high resolution radar
Ku-Band	glaciology (snow cover mapping)
Ka-Band	high resolution radar

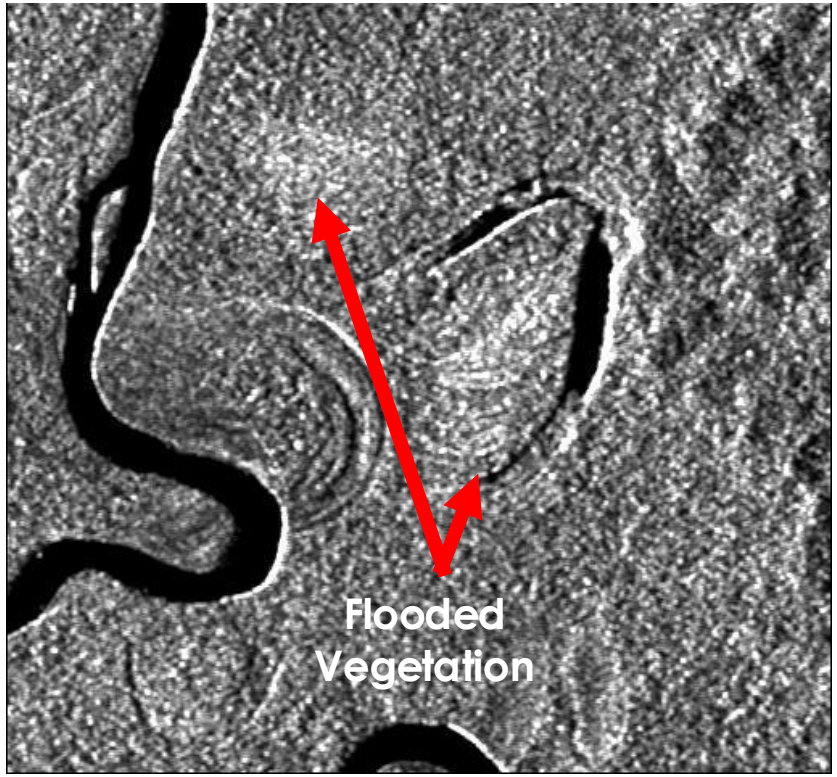
Example: Signal Penetration into Inundated Vegetation

Multi-frequency AIRSAR Data in Manu National Park, Peru

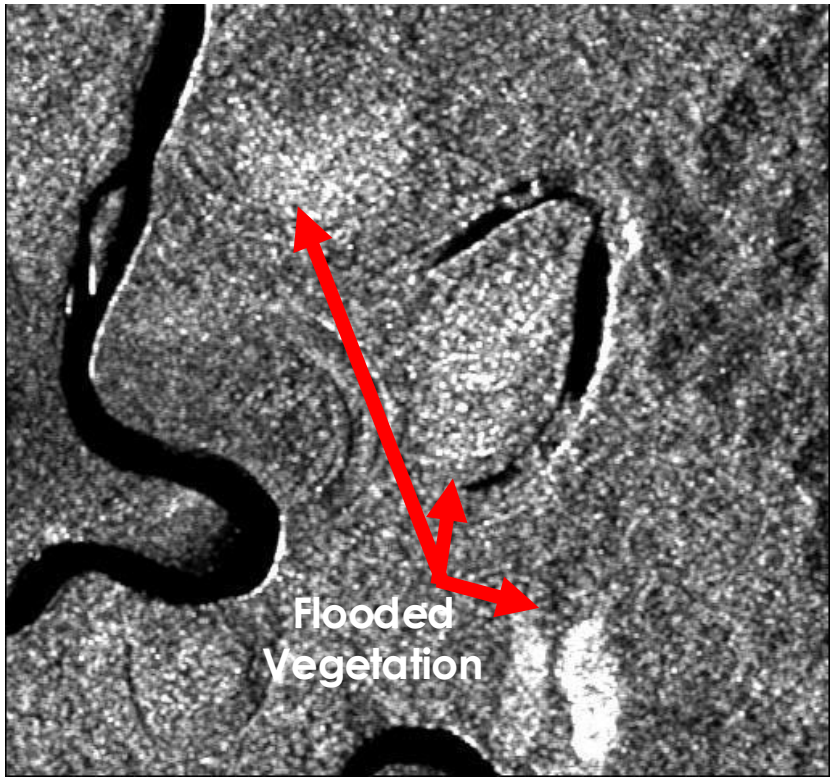
C-Band



L-Band



P-Band



Radar Parameter: Polarization

- **Polarization:** The orientation of the electric field of the electromagnetic wave.
- The polarizations are usually controlled between **H** and **V**:
  - HH: Horizontal Transmit, Horizontal Receive
  - VV: Vertical Transmit, Vertical Receive
  - HV: Horizontal Transmit, Vertical Receive
  - VH: Vertical Transmit, Horizontal Receive

- **Quad-Pol Mode:** When all four polarizations are measured.
- Different polarizations can determine physical properties of the object observed.

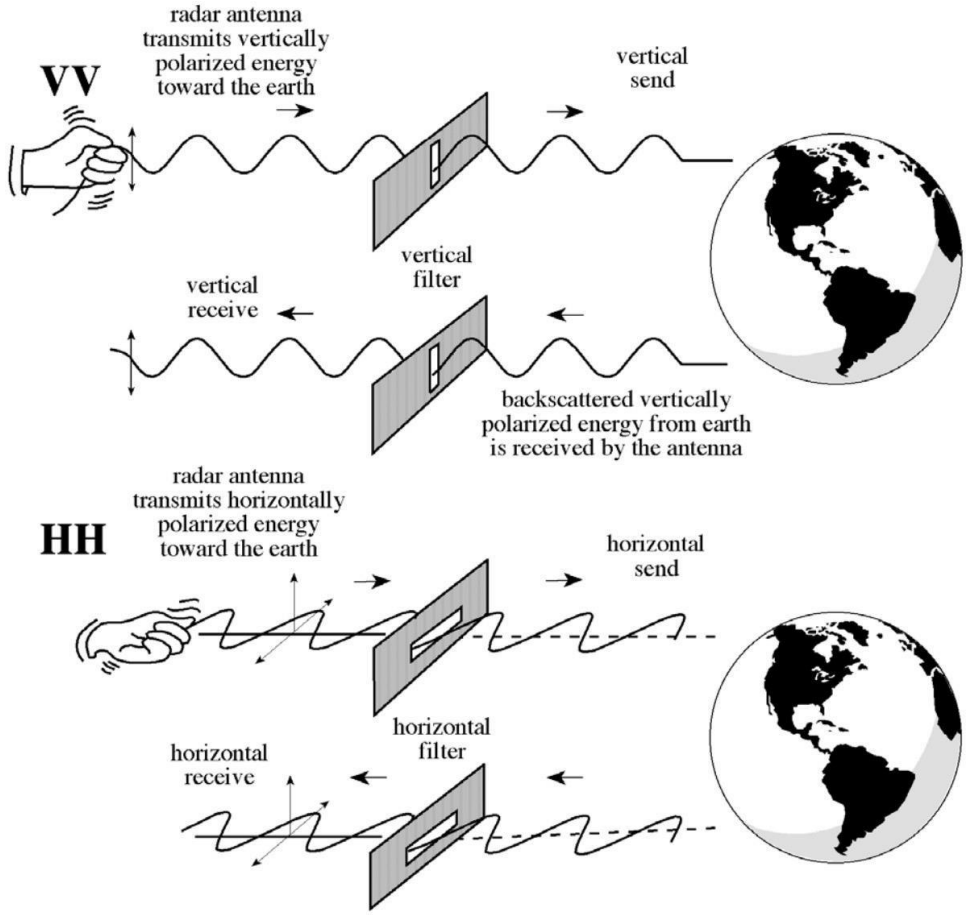
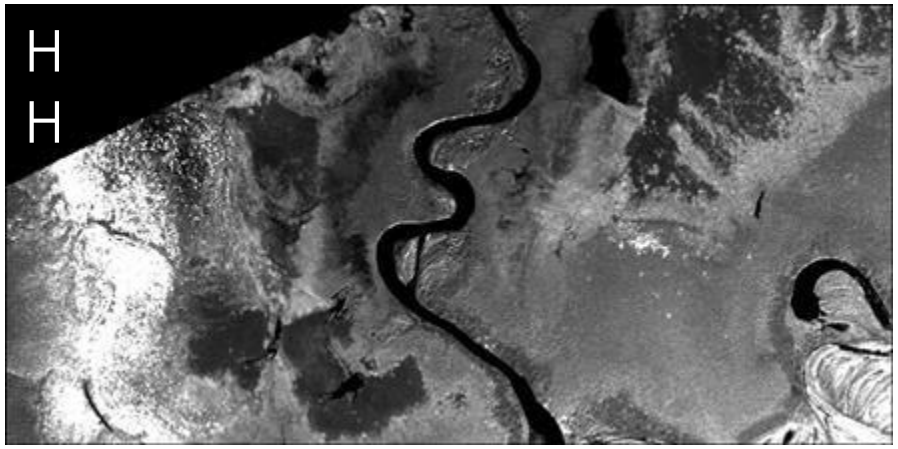


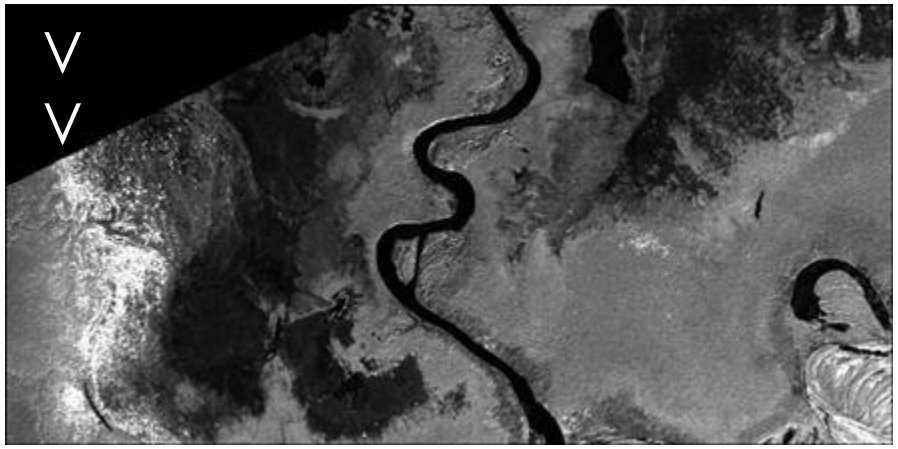
Image Credit: J.R. Jensen, 2000. Remote Sensing of the Environment

Example of Multiple Polarizations for Vegetation Studies

Pacaya-Samiria Forest Reserve in Peru

Images from UAVSAR (HH, HV, VV)



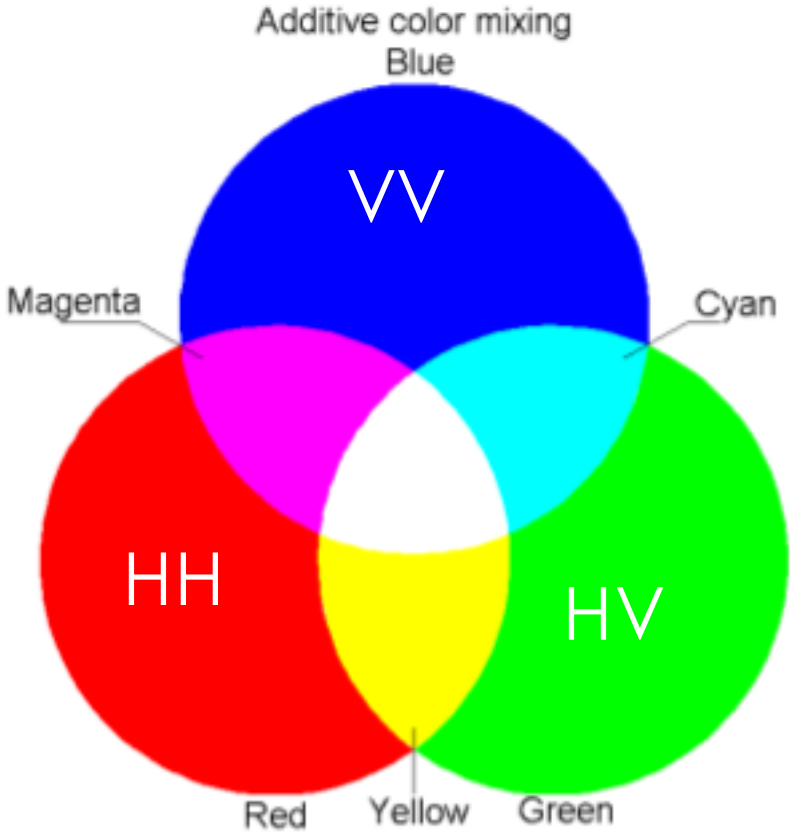


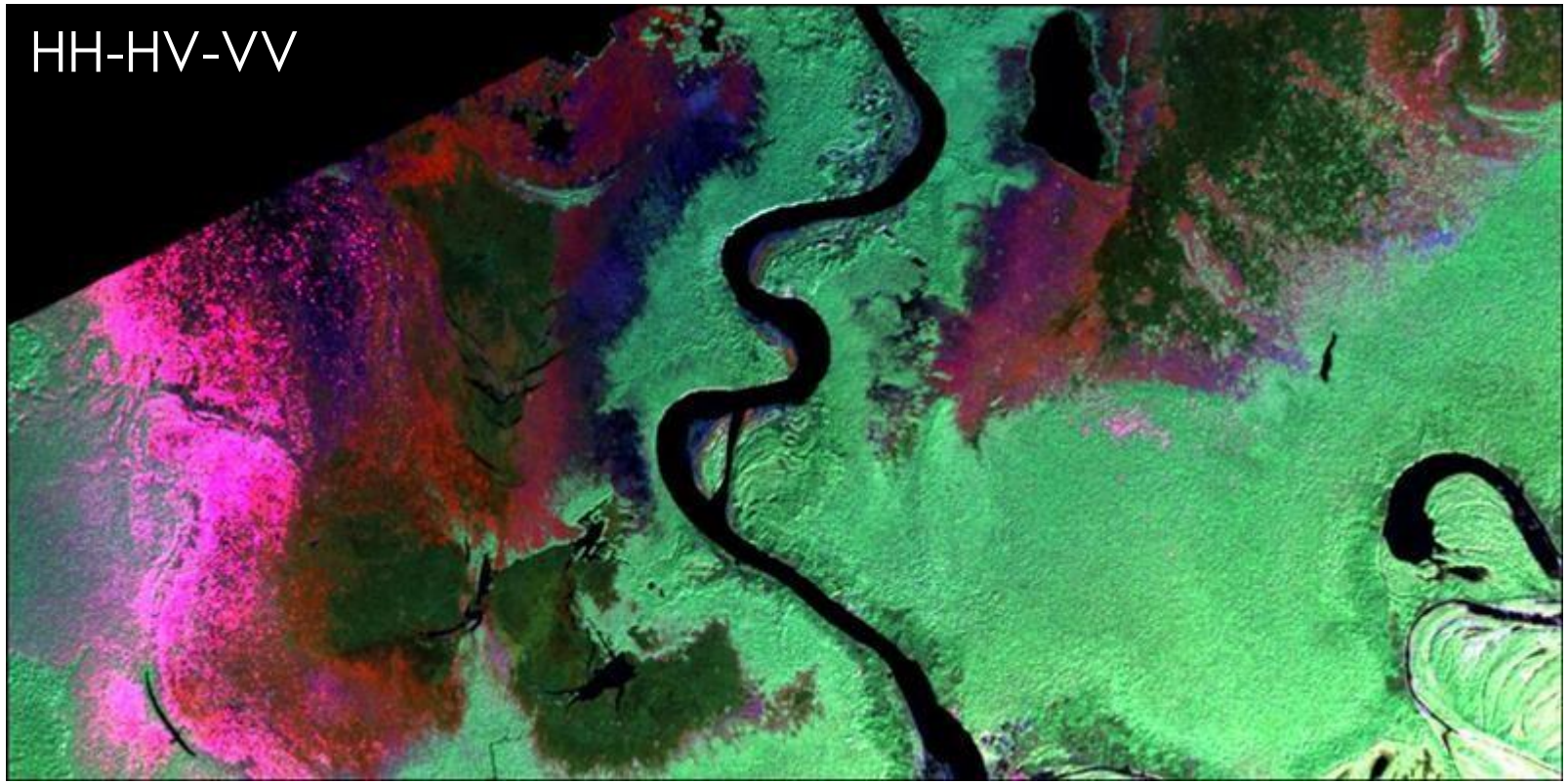


Visualization of Multiple Polarizations for Vegetation Studies

Pacaya-Samiria Forest Reserve in Peru

Images from UAVSAR (HH, HV, VV)





Example: Signal Penetration into Vegetation

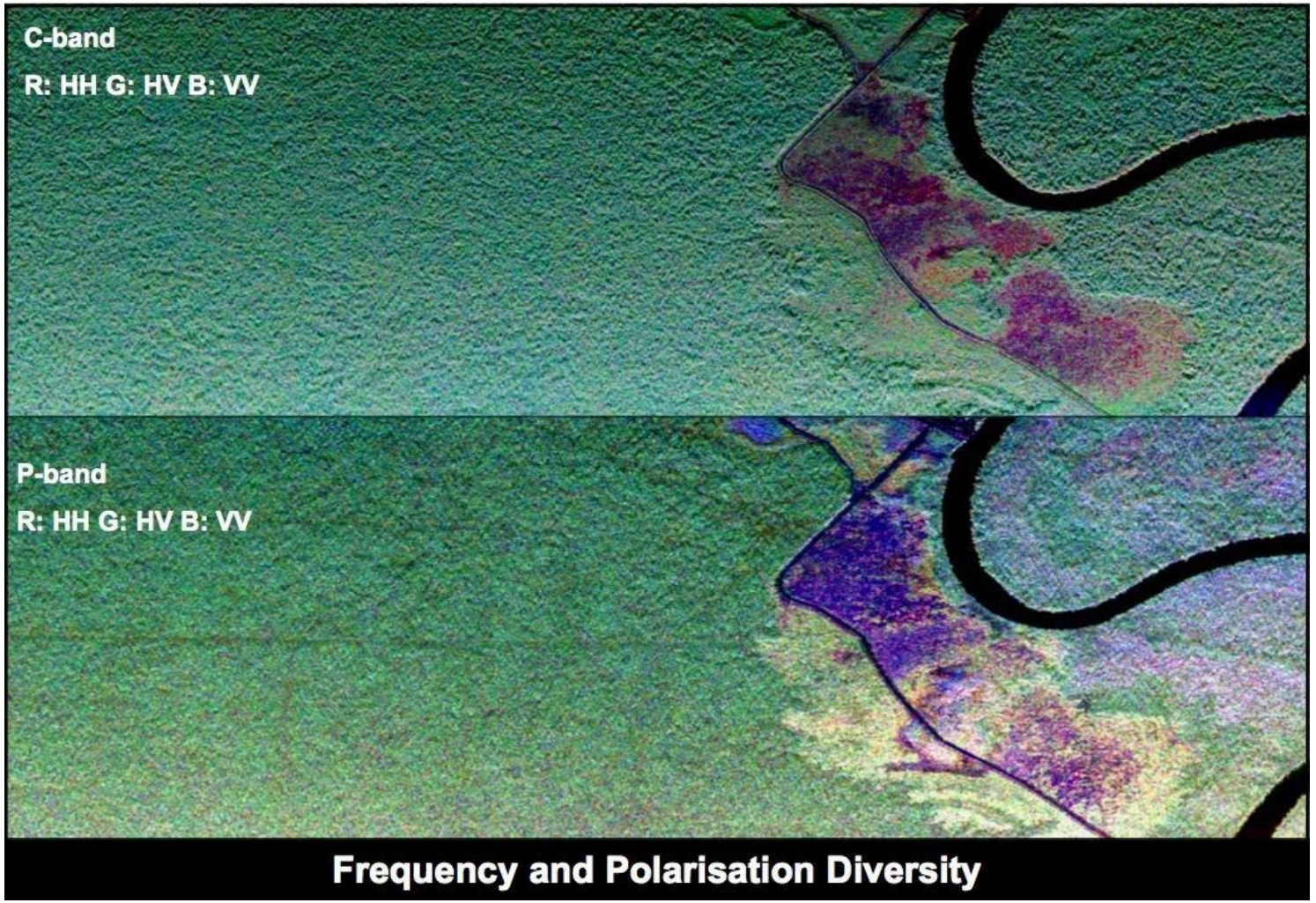
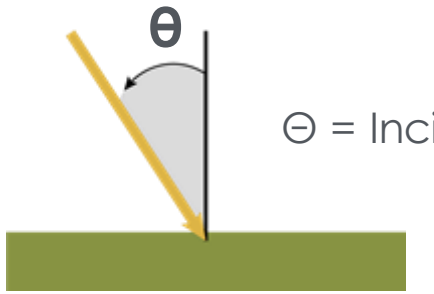


Image Credit: A Moreira (ESA)

Radar Parameter: Incidence Angle

- The angle between the direction of illumination of the radar and the Earth's surface plane.
- Is dependent on the height of the sensor.
- Influences image brightness.
- This is why the geometry of an image is different from point to point in the range direction.



Θ = Incidence Angle

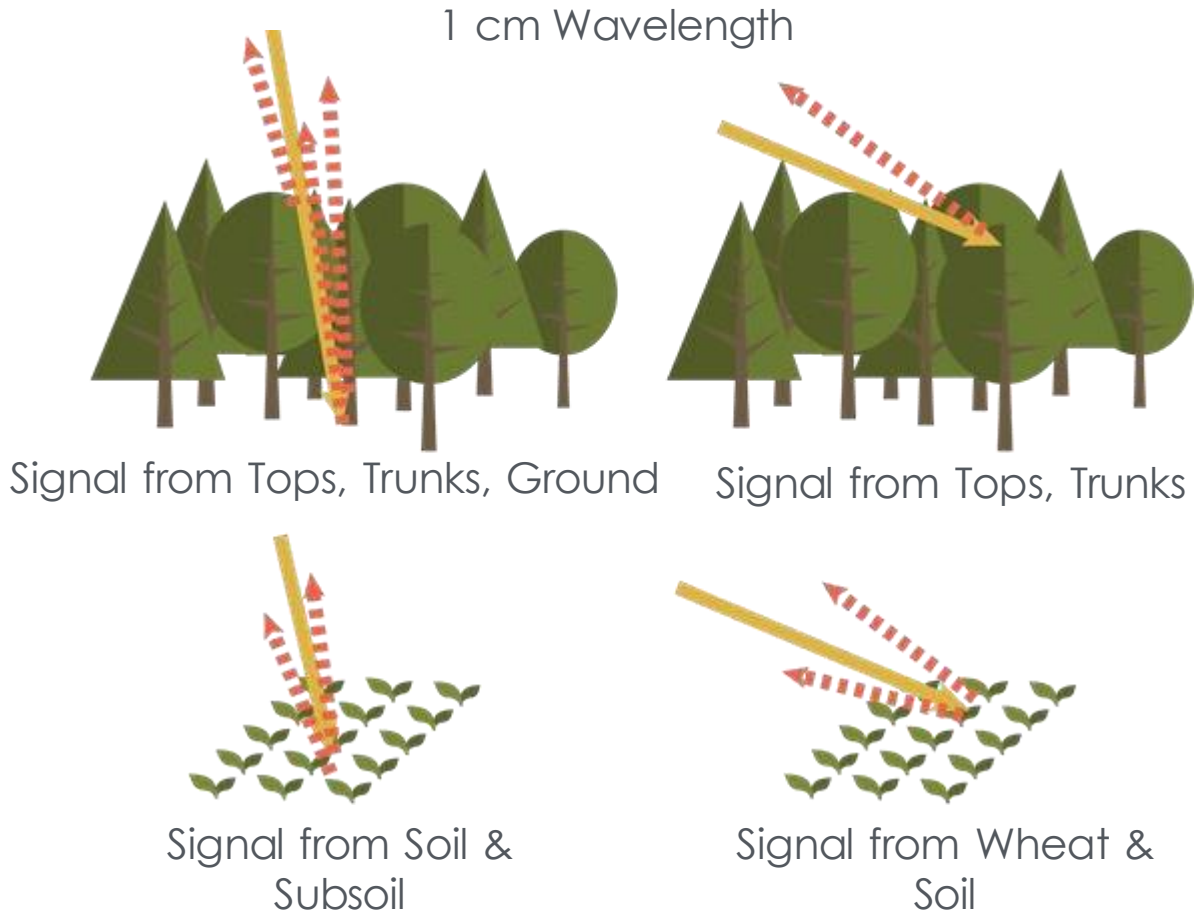
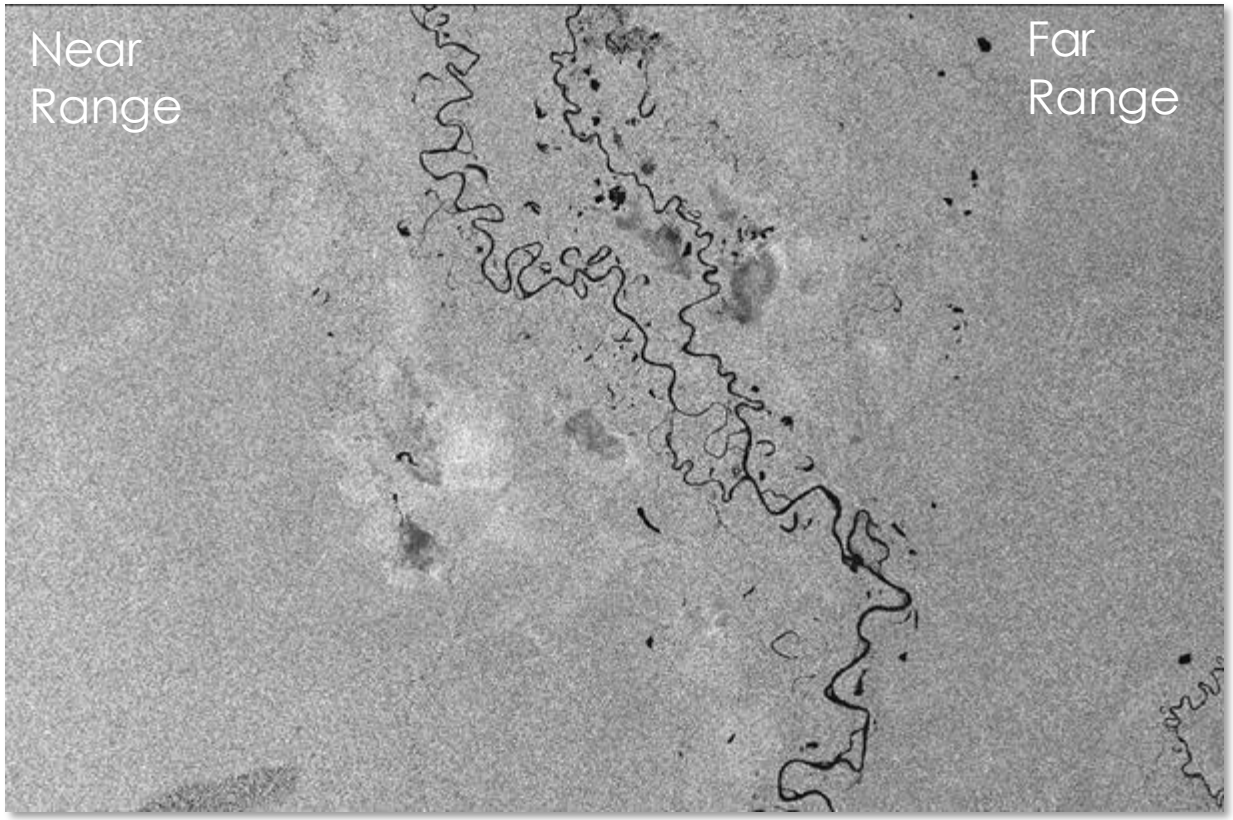
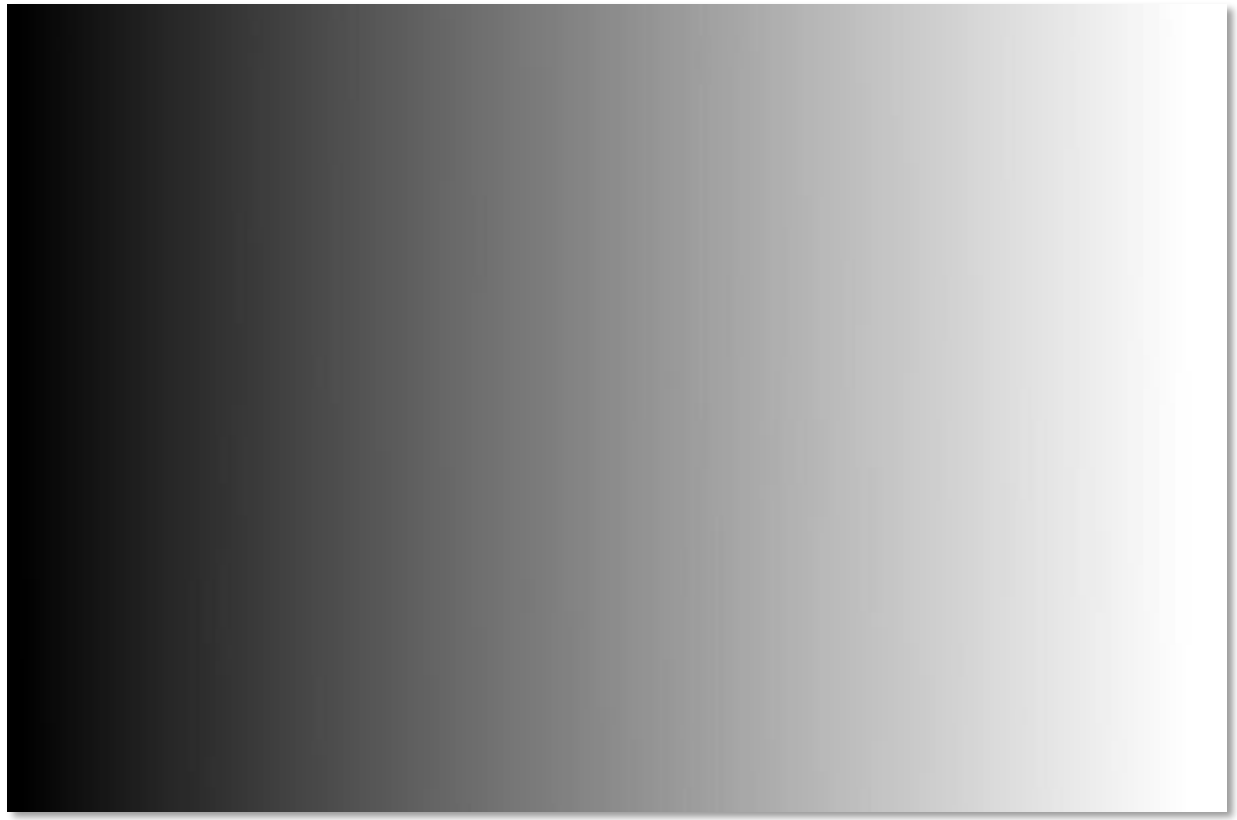


Image Credit: Ulaby et al. (1981a) and ESA

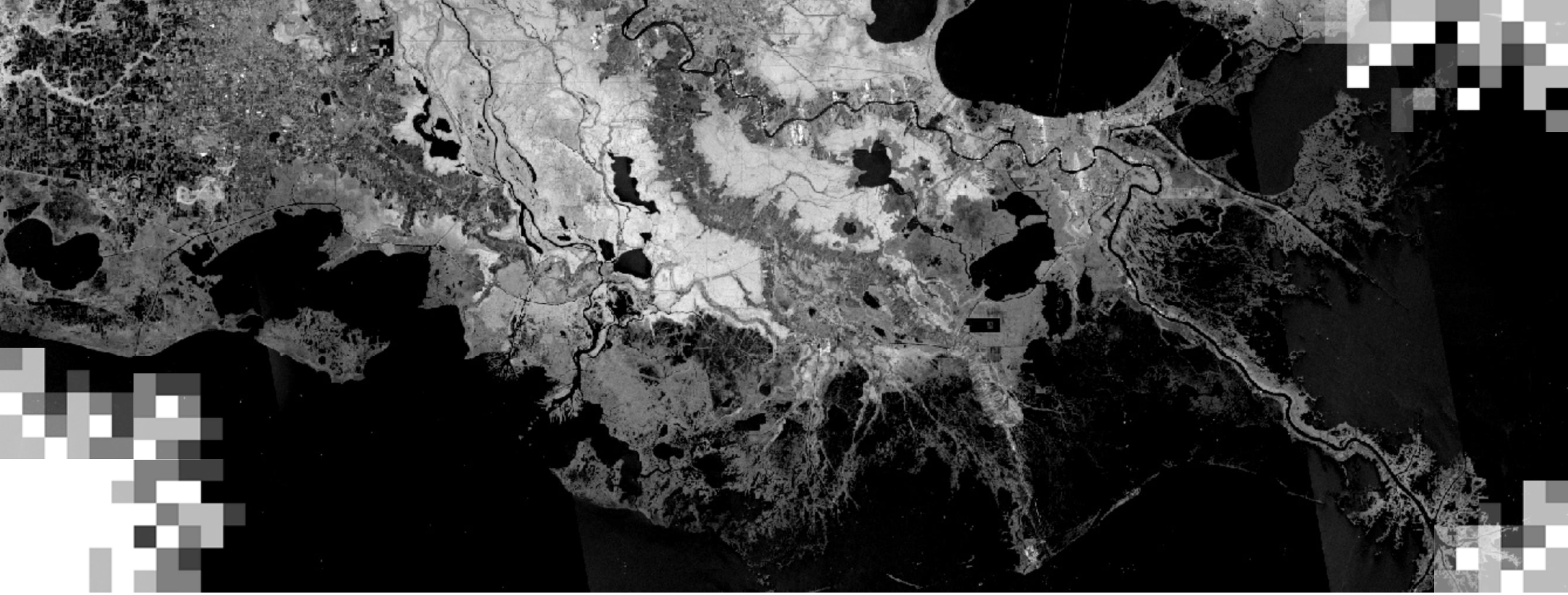
Effect of Incidence Angle Variation



Sentinel-1



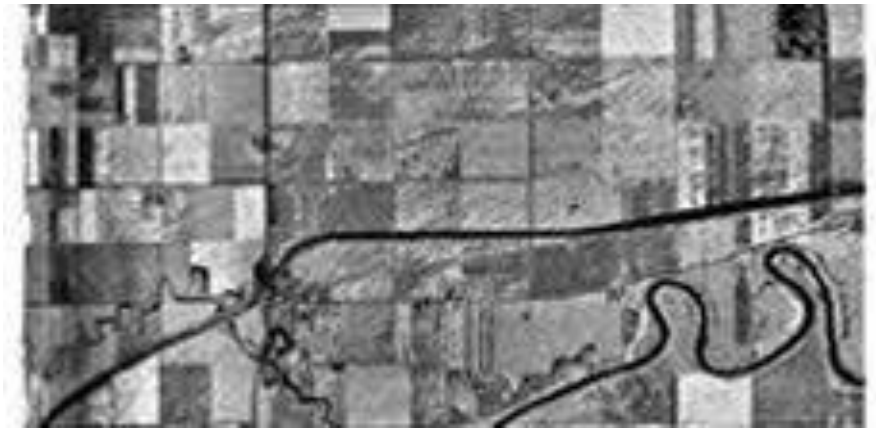
30 Incidence Angle (Degrees) 45



Radiometric and Geometric Distortions

Slant Range Distortion

Slant Range



Ground Range

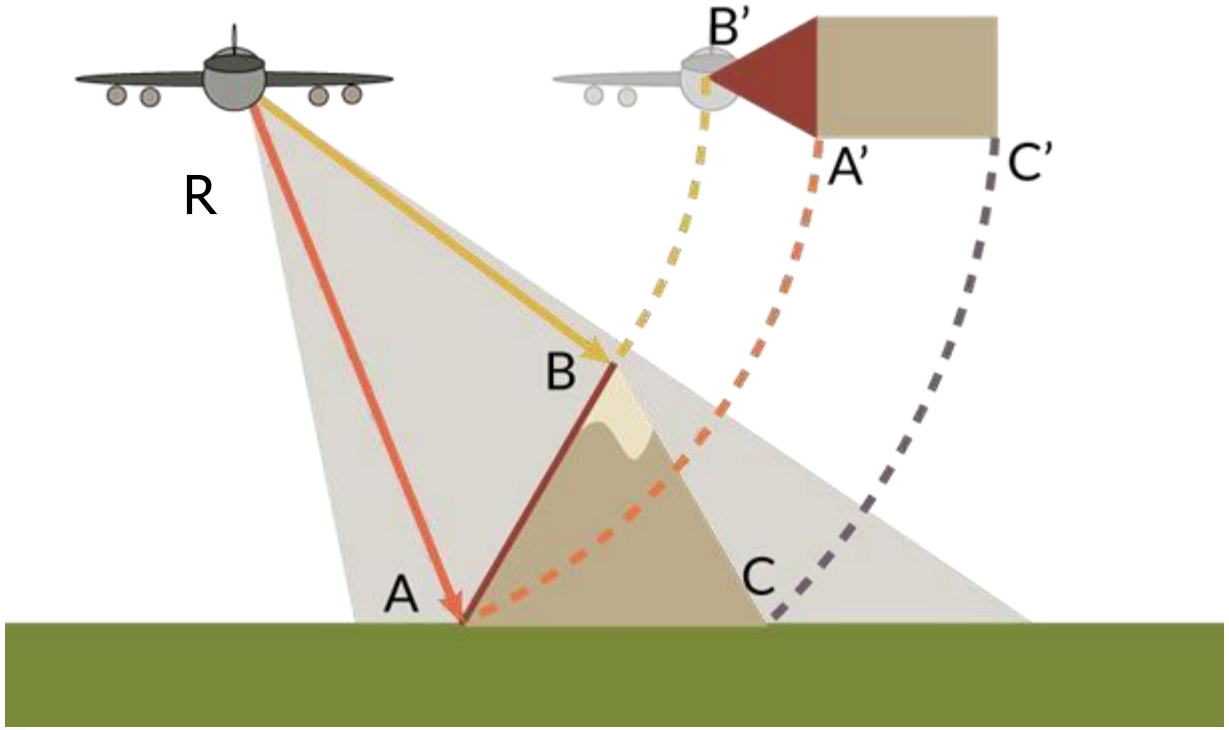


Source: Natural Resources Canada

Geometric Distortion: Layover and Foreshortening

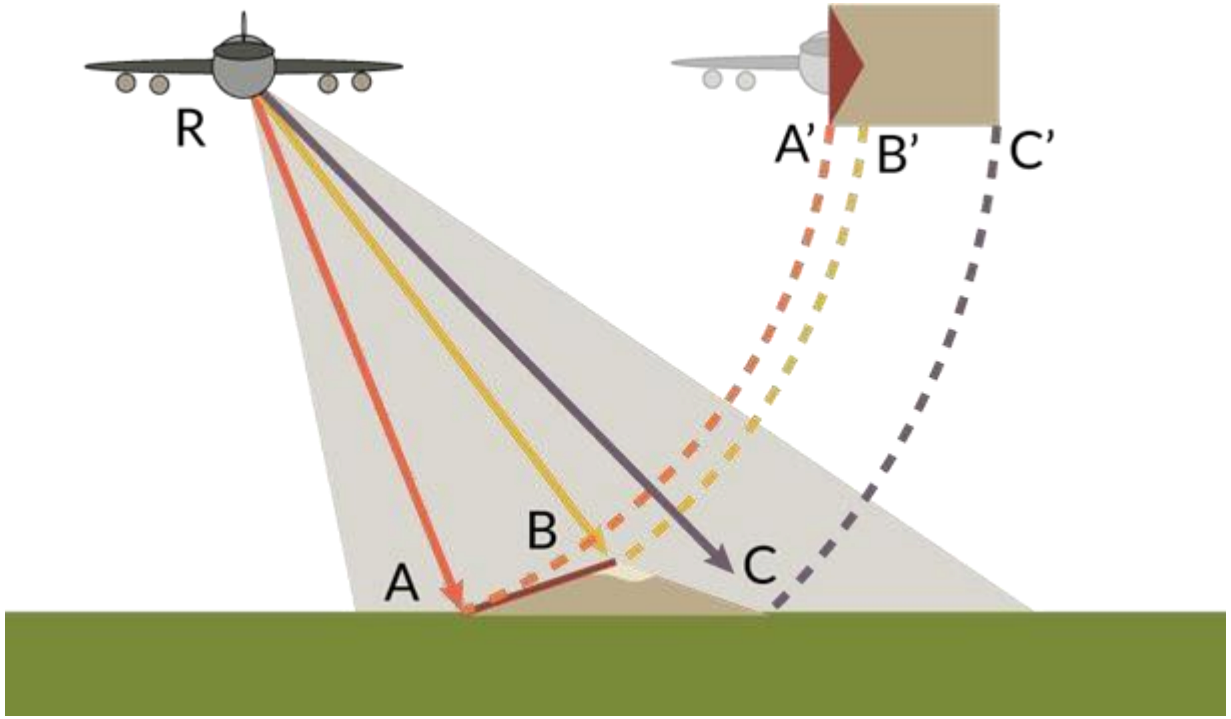
Layover

AB = BC
A'B' < B'C'
RA > RB
RA' > RB'



Foreshortening

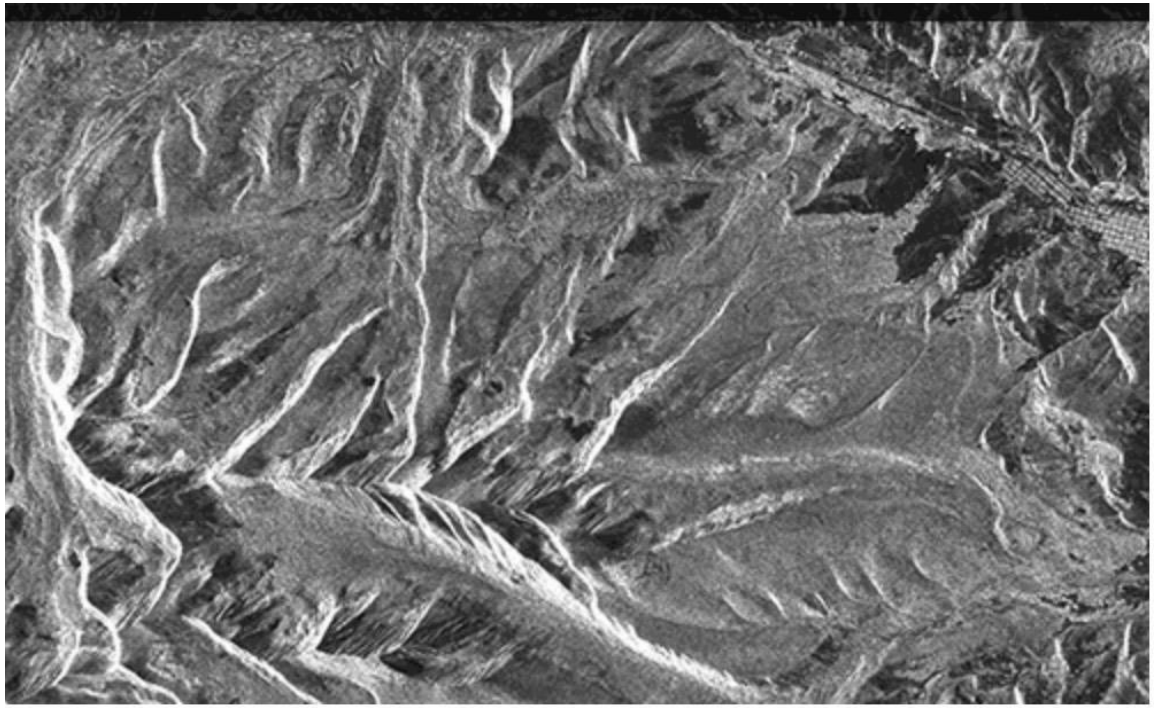
RA < RB < RC
AB = BC
A'B' < B'C'



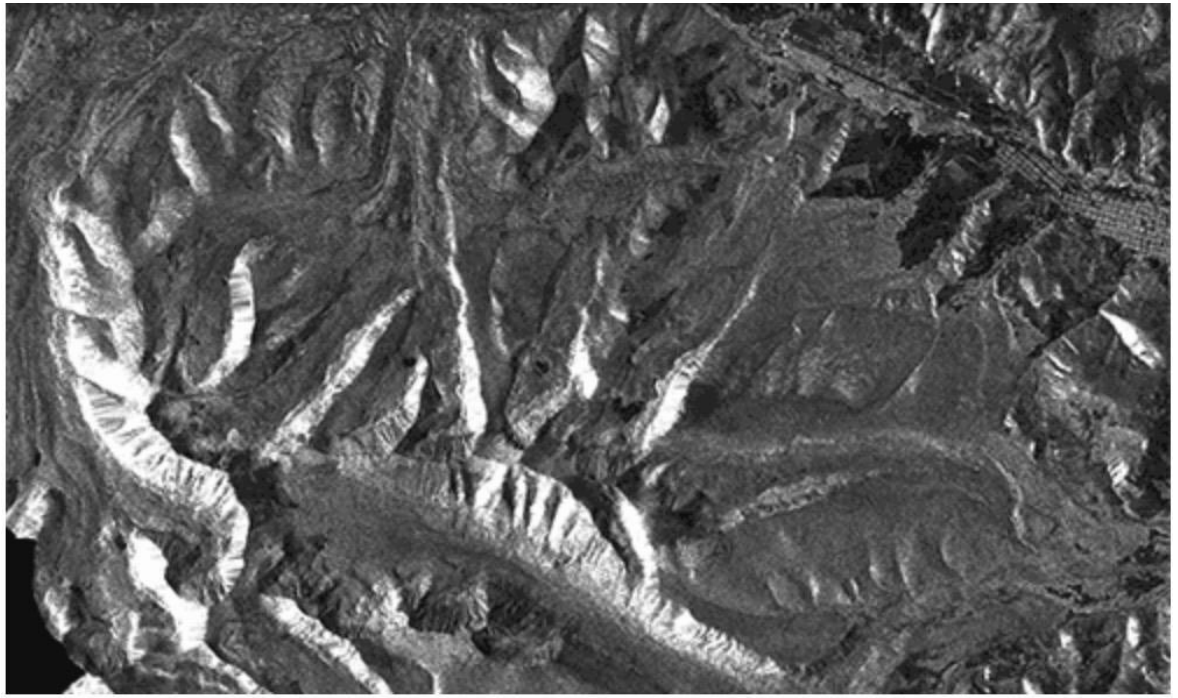
Source: Natural Resources Canada

Foreshortening

Before Correction

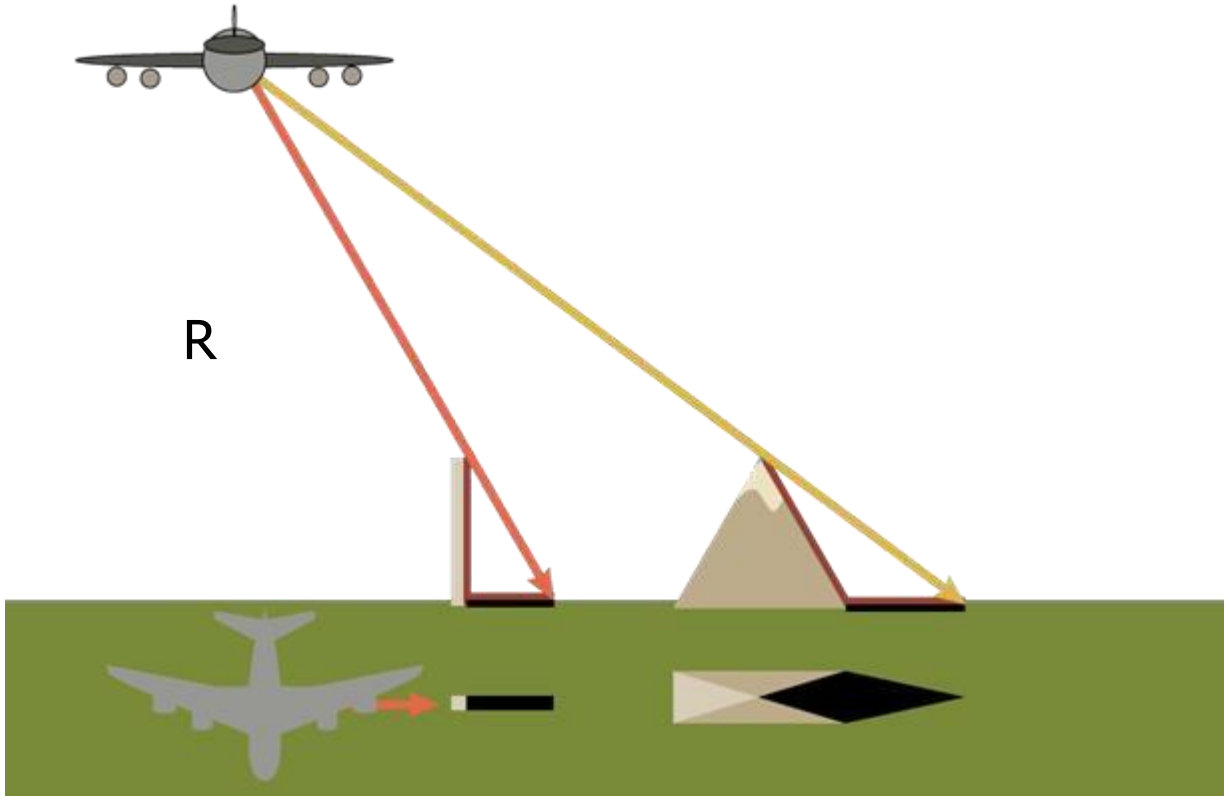


After Correction



Source: ASF

Shadow



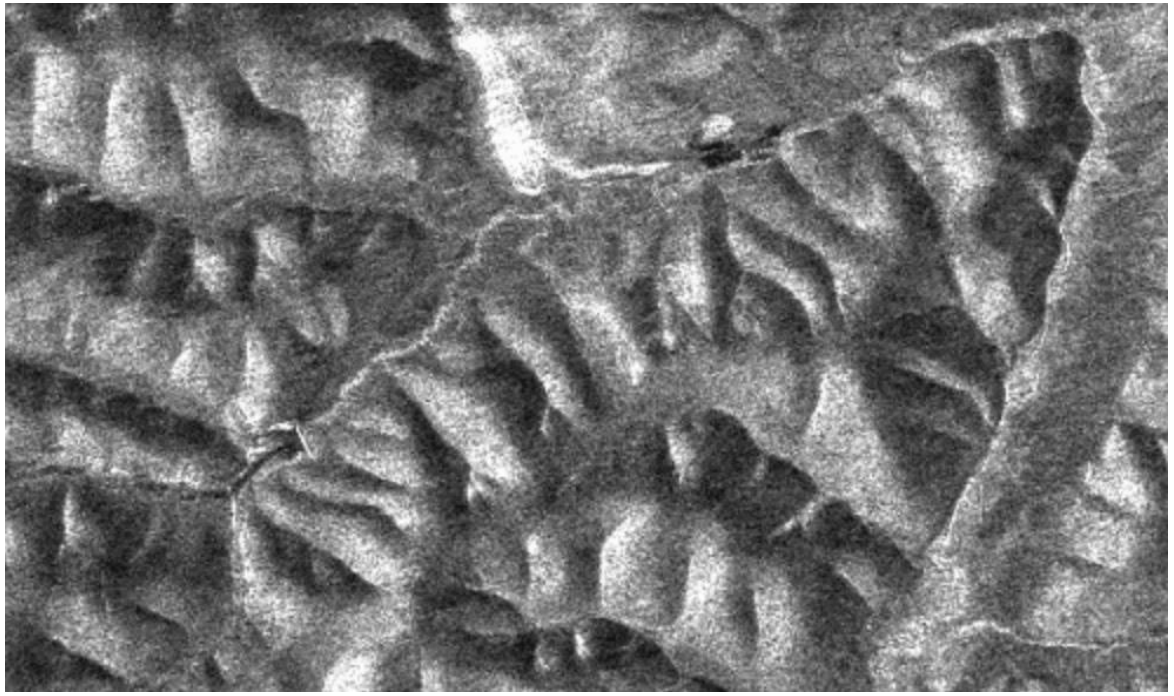
Source: Natural Resources Canada



Radiometric Distortions

- The influence of topography on backscatter should be corrected.
- This correction eliminates high values in areas of complex topography.

Before Correction



After Correction

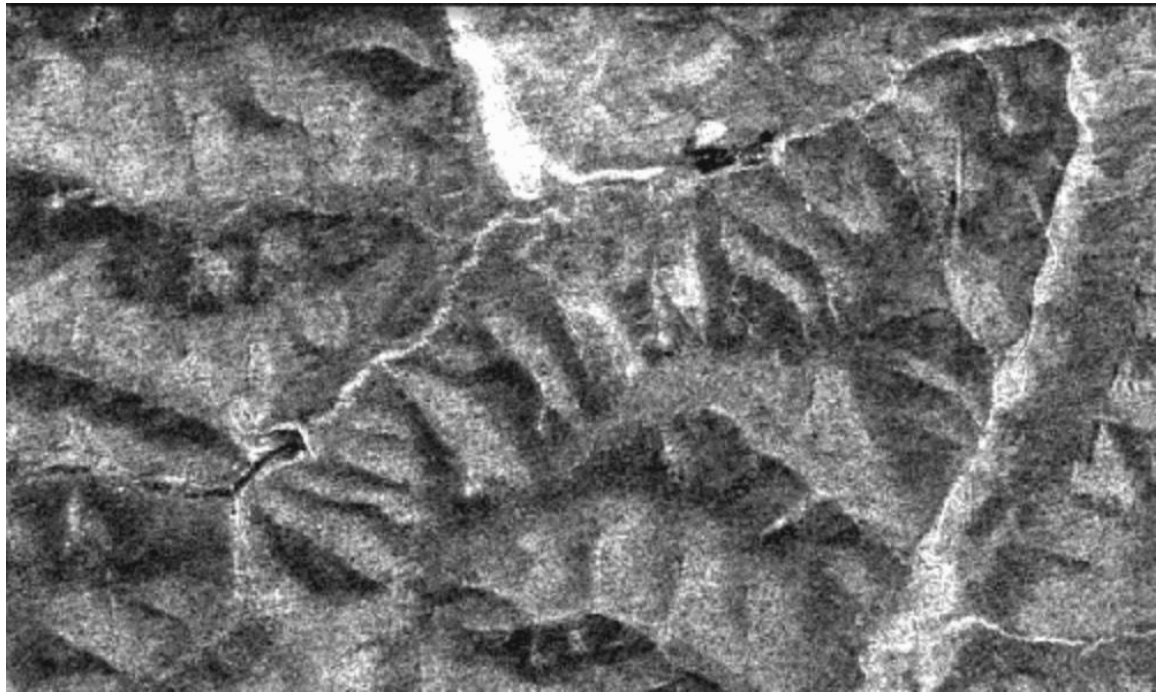
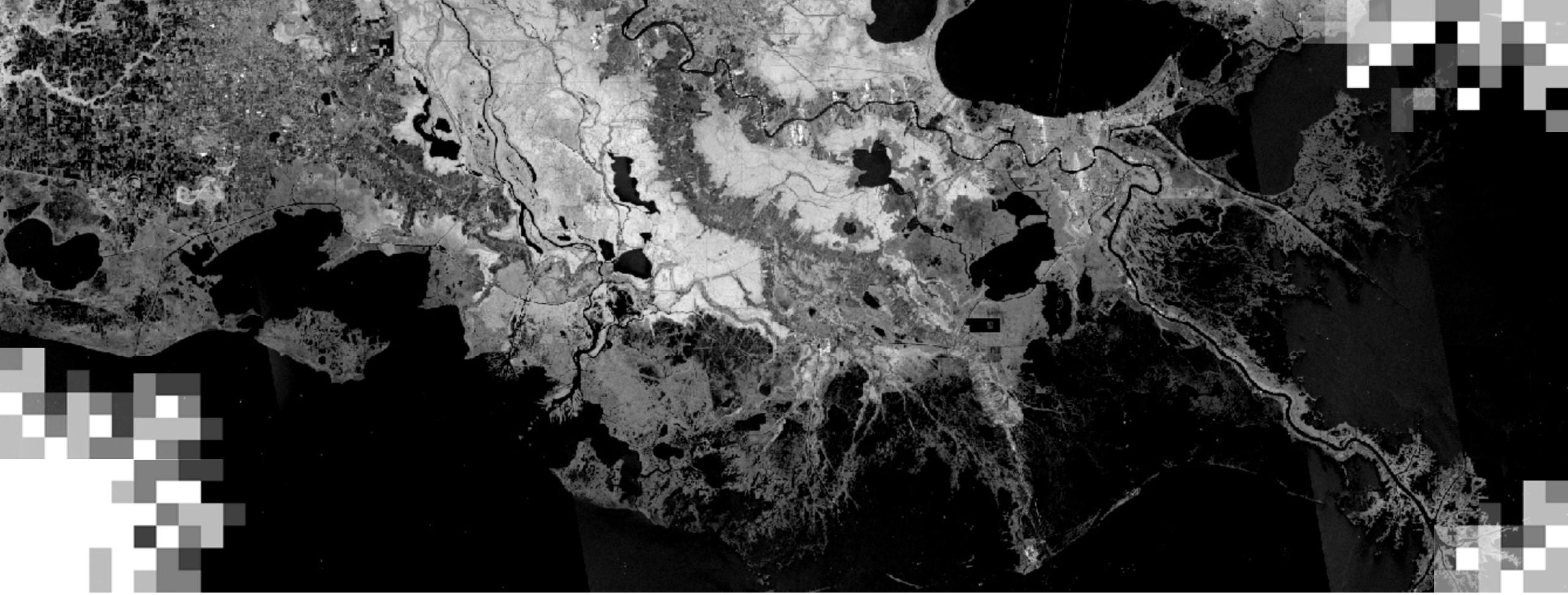


Image Credit: ASF



Speckle

Speckle

- **Speckle** is a granular 'noise' that inherently exists in and degrades the quality of SAR images.

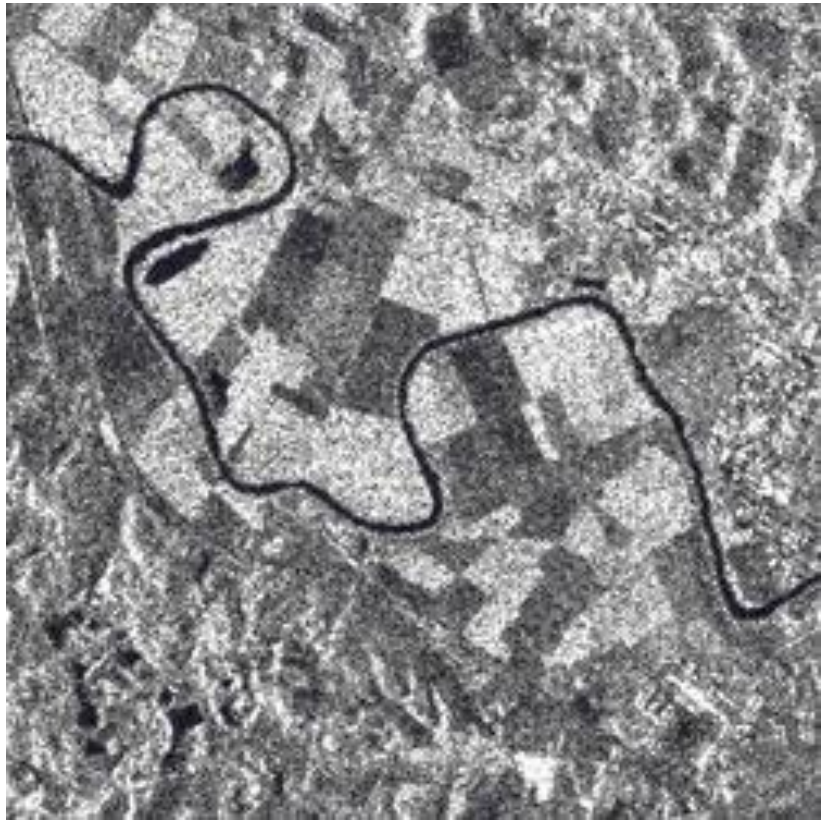


Image Credit: ESA

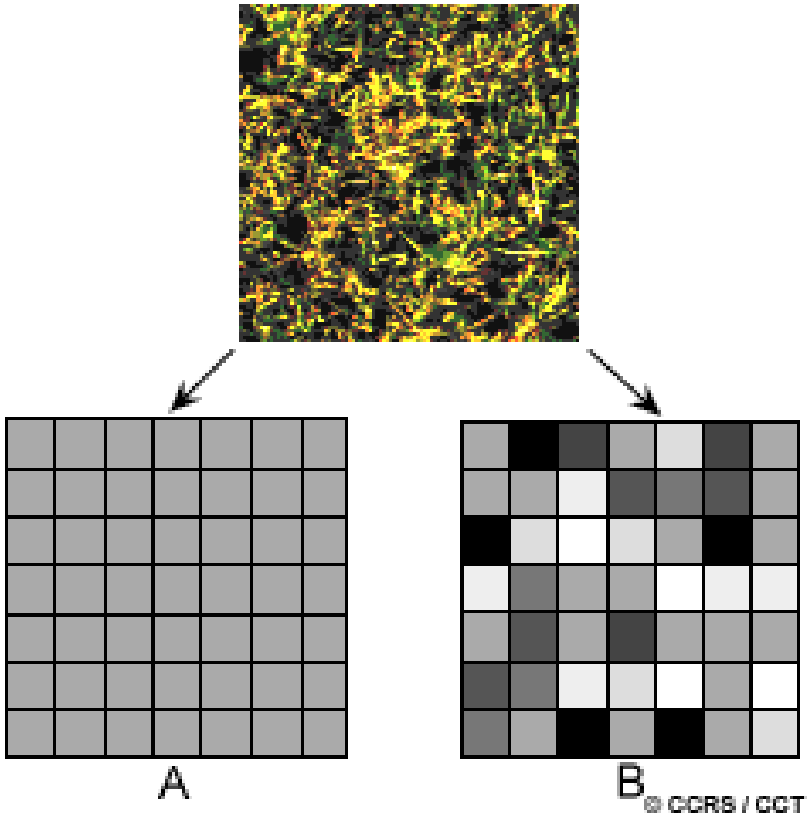
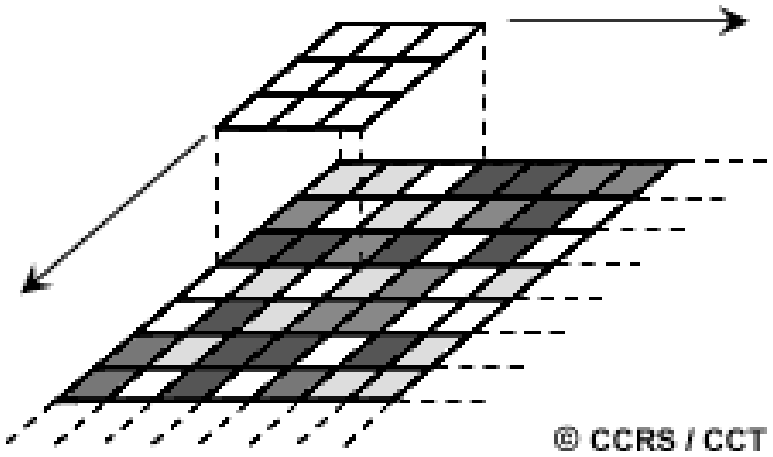


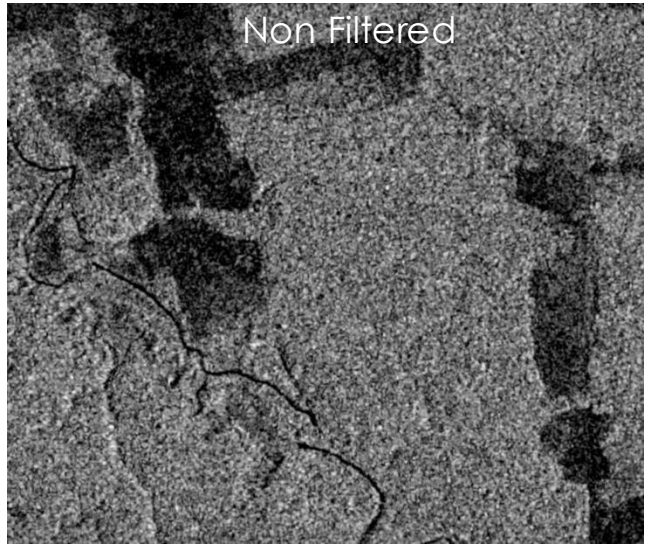
Image Credit: Natural Resources Canada

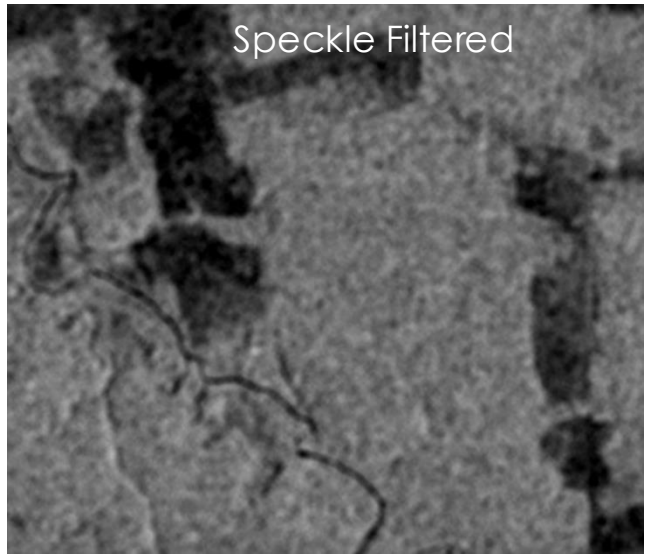
Speckle Reduction: Spatial Filtering

- Moving window over each pixel in the image
- Applies a mathematical calculation on the pixel values within the window
- The central pixel is replaced with the new value
- The window is moved along the x and y dimensions one pixel at a time
- Reduces visual appearance of speckle and applies a smoothing effect



Source: Natural Resources Canada





Synthetic Aperture Radar

Azimuth Resolution (Along Track)

- It is difficult to have long antennas in space, but one can be synthesized by using the movement of the satellite to simulate a very long antenna.

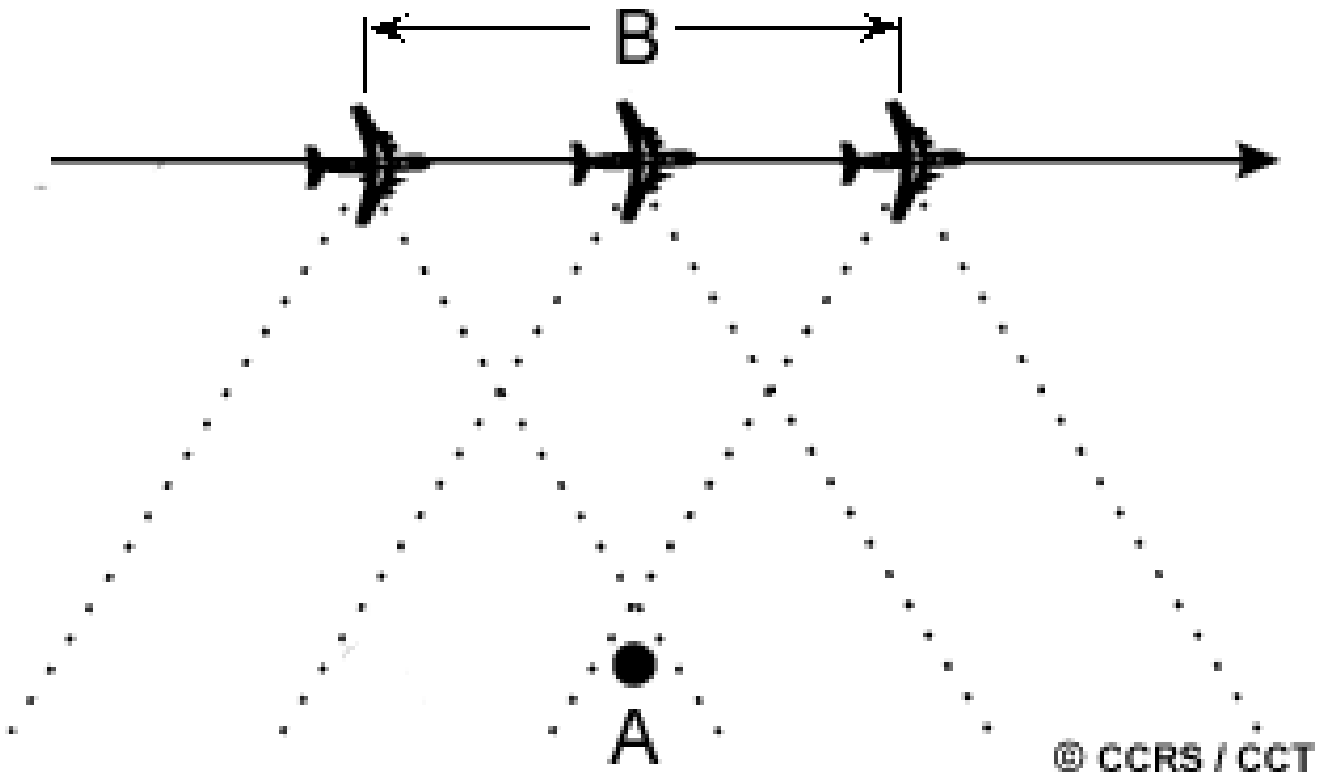
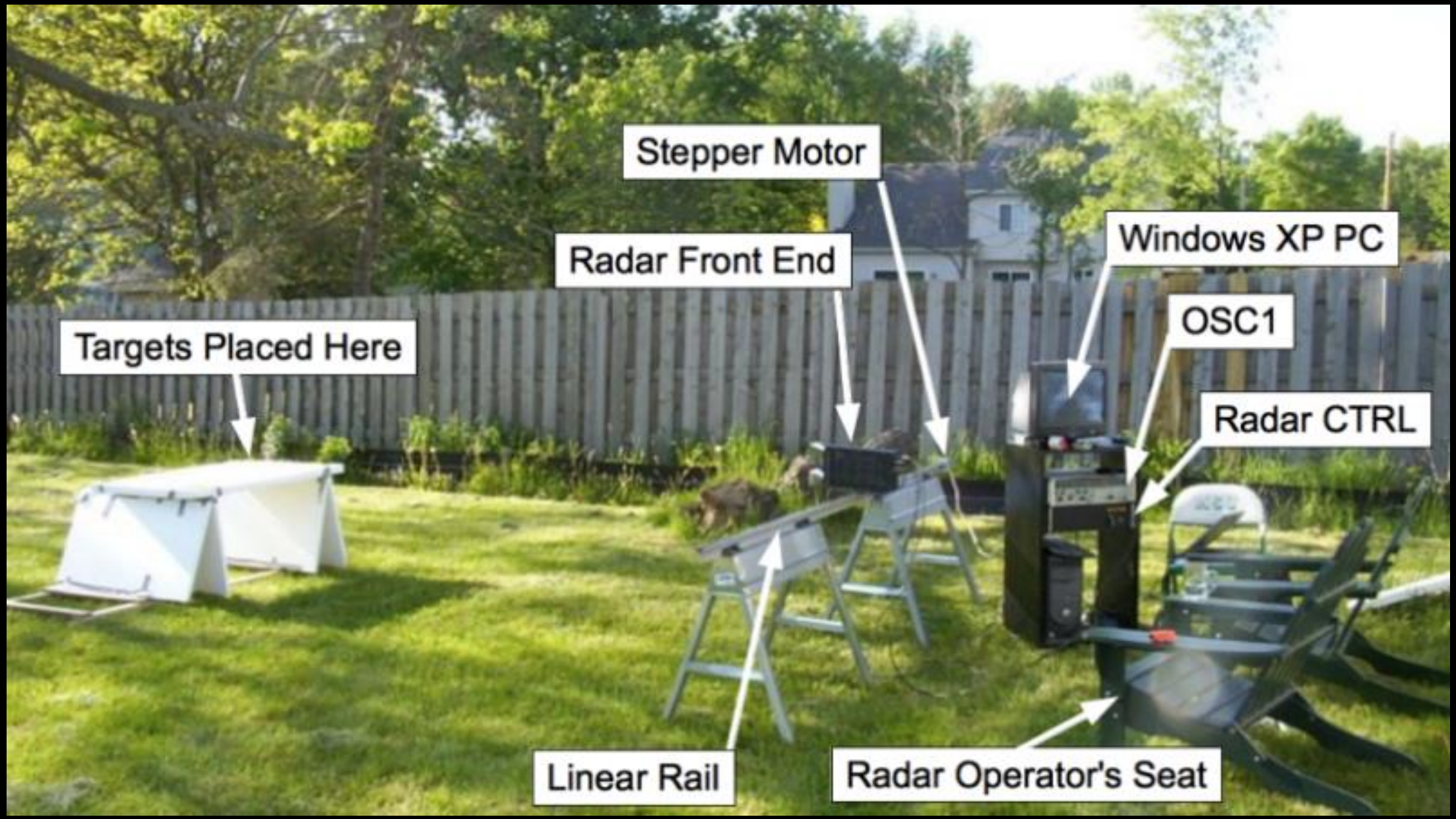


Image Credit: Lillesand and Kiefer, 7th Edition



Source: https://hackaday.com/2014/03/17/radar-imaging-in-your-garage-synthetic-aperture-radar/

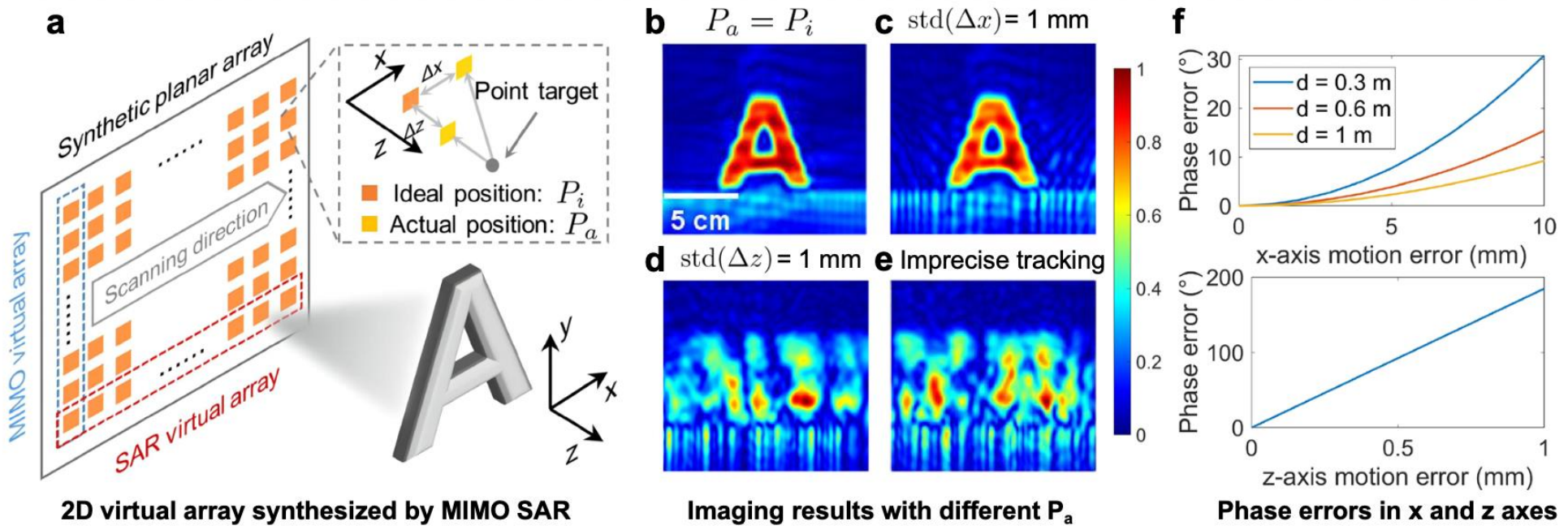
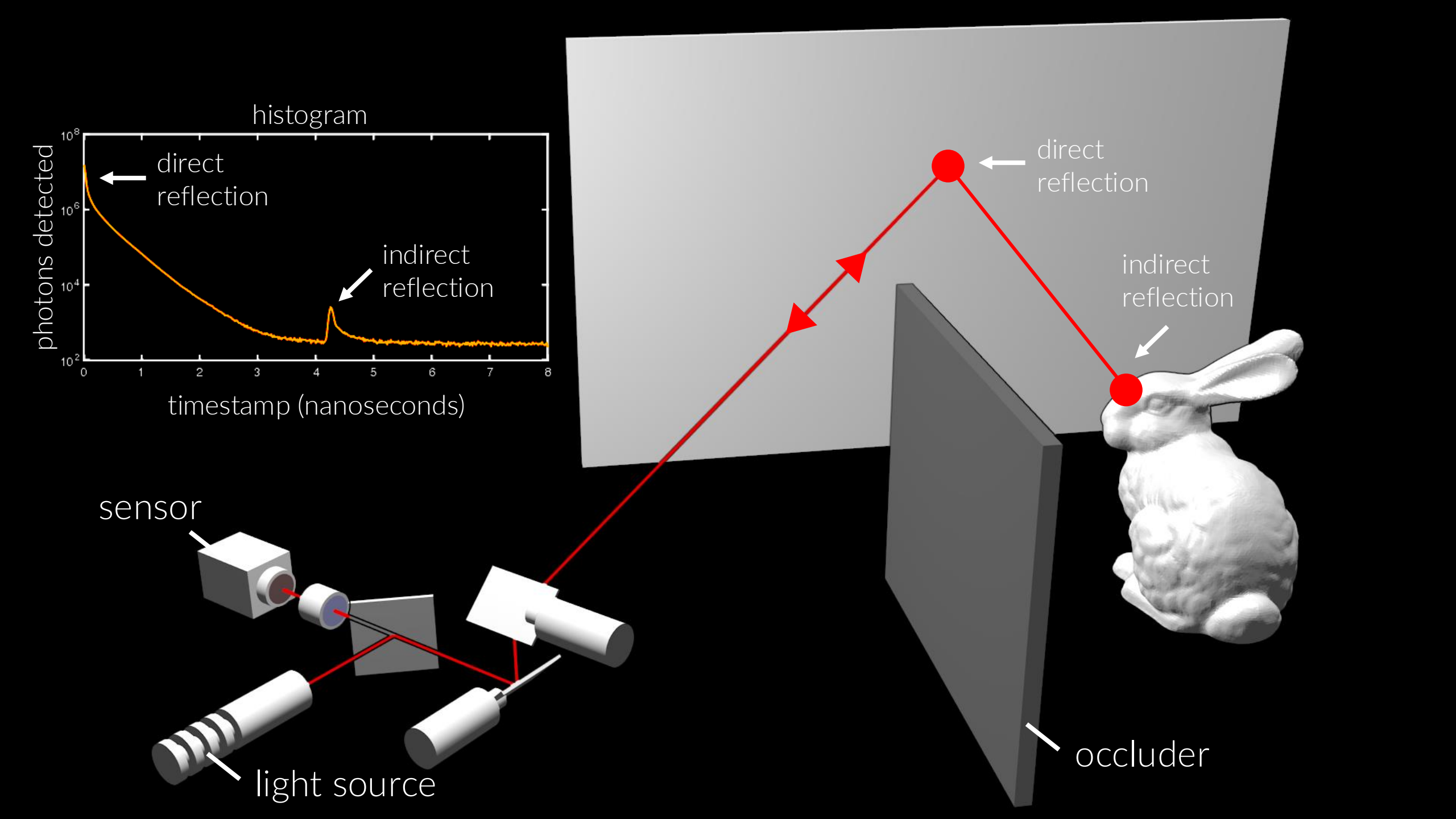


Fig. 1 The spatial asymmetry of phase errors. a An ideal planar aperture can be synthesized by mechanically moving a linear multi-input-multi-output (MIMO) array. In handheld synthetic aperture radar (SAR) imaging, the actual positions of the antenna will deviate from the ideal positions. In the established Cartesian coordinate system, the x-axis, y-axis, and z-axis denote the horizontal scanning, vertical, and depth directions, respectively. Δx and Δz denote the motion error along the x-axis and z-axis, respectively. **b** The imaging result obtained using time-domain back-projection[53] with the ideal antenna positions, which appears clear and well-defined (see the experimental setup in Supplementary Note 1). **c**, **d** These demonstrate the imaging results when statistical position errors with a standard deviation (std) of 1 mm (see Supplementary Fig. 1a, b) are added to the x-axis and z-axis, respectively. The z-axis position errors introduce the image's most significant distortions and blurring. **e** The blurred image is generated by the recorded positions of a tracking camera (see Supplementary Fig. 1c), showcasing the impact of imprecise localization. **f** The z-axis motion errors result in much larger phase errors than the x-axis and y-axis motion errors, where *d* denotes the z-axis distance between the target and the antenna. Overall, these results highlight the importance of accurate position estimation in handheld SAR imaging, particularly in mitigating z-axis motion errors due to the asymmetry of phase errors along different directions.

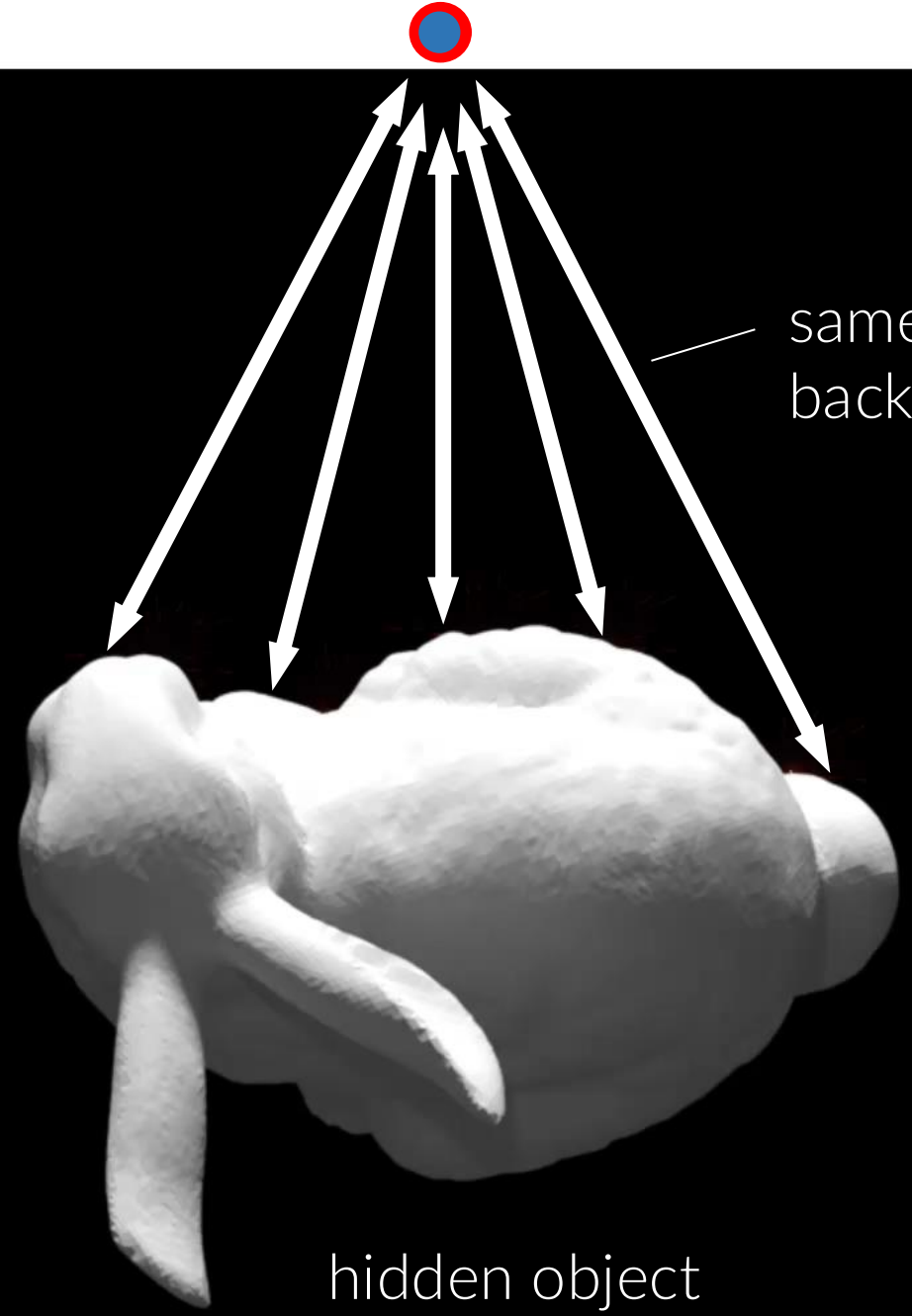
TI four-chip cascaded mmWave radar

Li et al., "A high-resolution handheld millimeter-wave imaging system with phase error estimation and compensation" [2024]

Back to Optical NLOS Imaging


histogram
photons detected
10^8
10^6
10^4
10^2
0
1
2
3
4
5
6
7
8
timestamp (nanoseconds)
direct
reflection
indirect
reflection
direct
reflection
indirect
reflection
sensor
light source
occluder

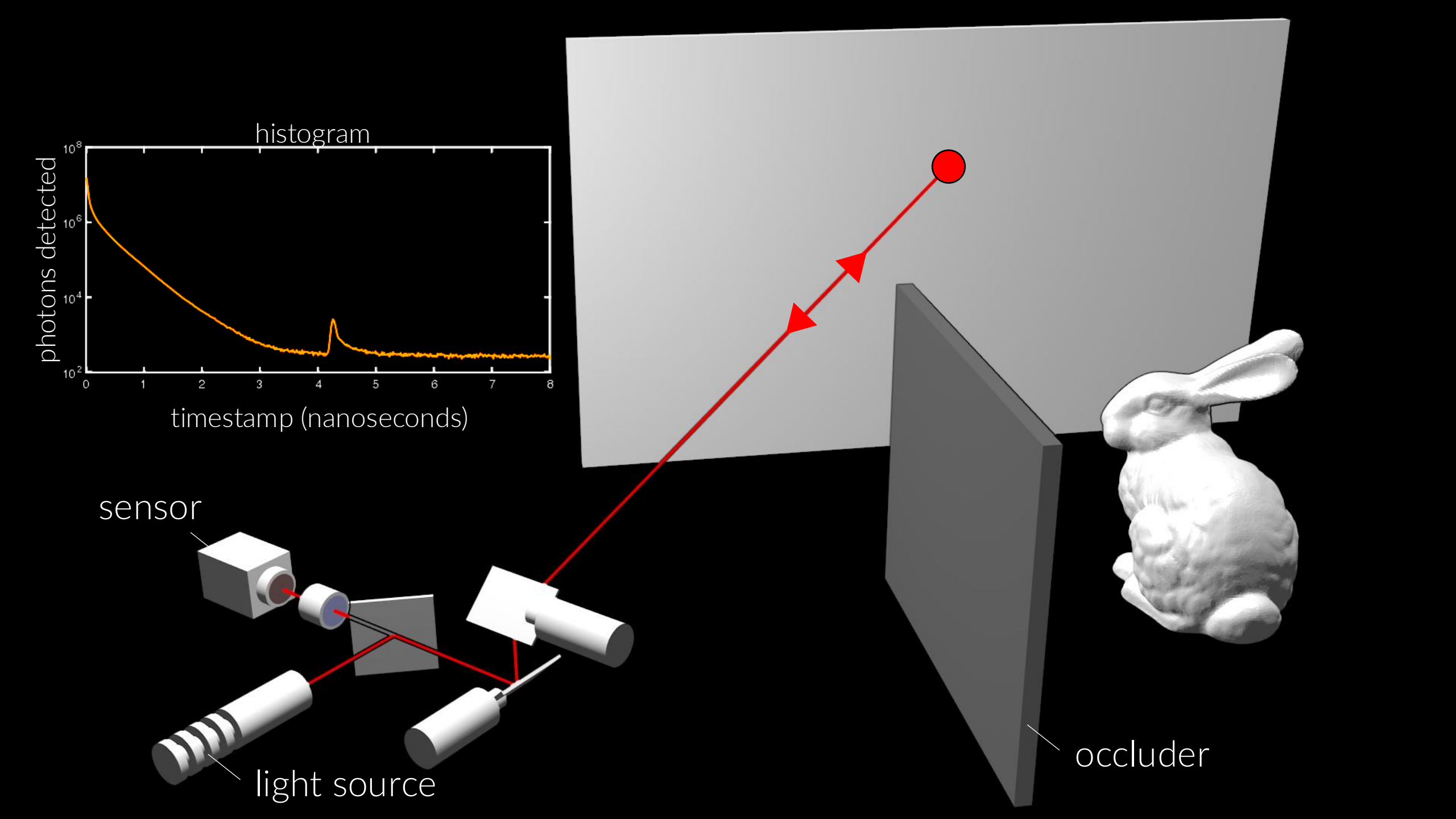
wall

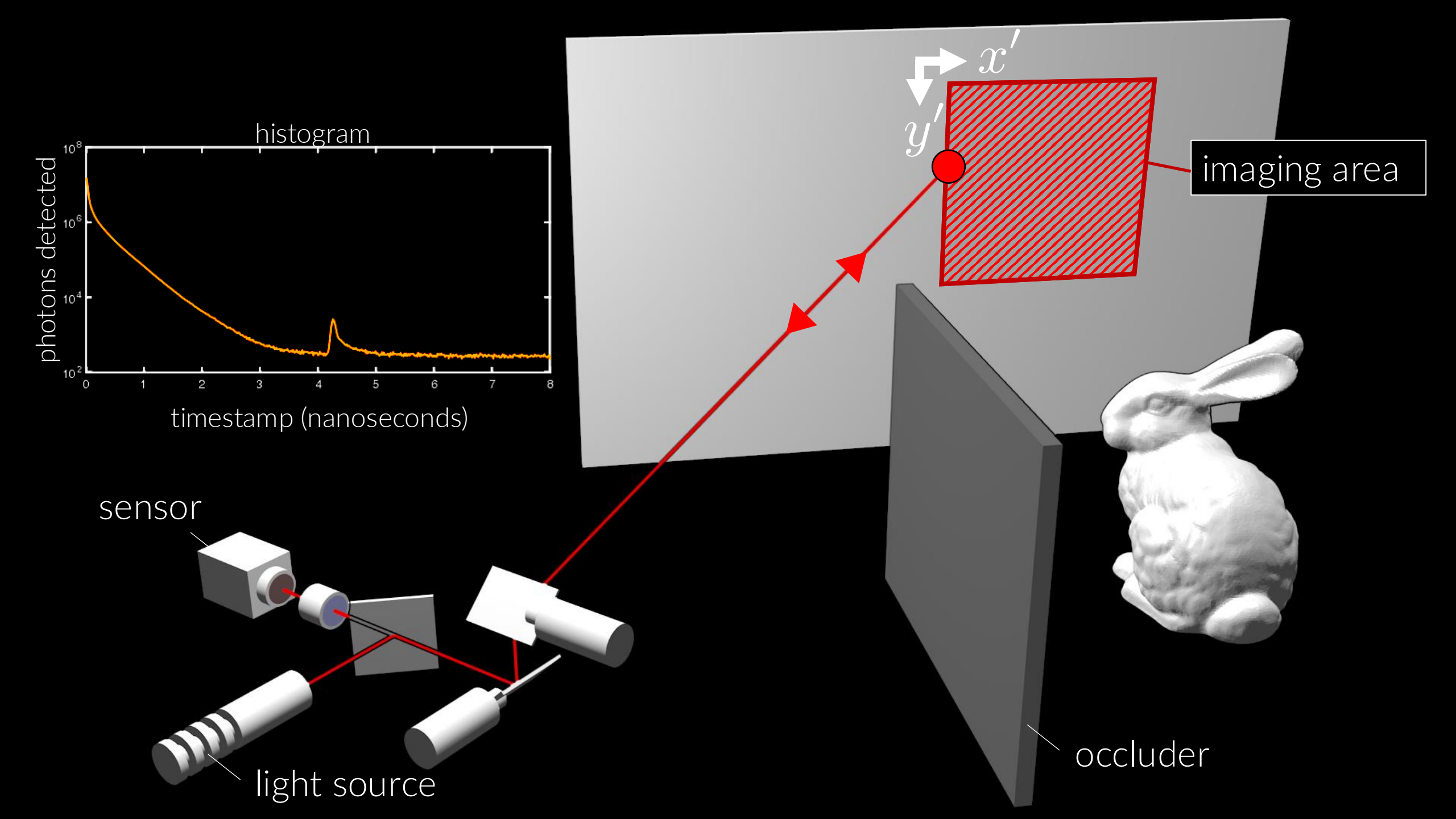


same path to the object and back

hidden object

confocal sampling


histogram
photons detected
10^8
10^6
10^4
10^2
0
1
2
3
4
5
6
7
8
timestamp (nanoseconds)
sensor
light source
occluder


histogram
photons detected
10^8
10^6
10^4
10^2
0
1
2
3
4
5
6
7
8
timestamp (nanoseconds)
x'
y'
imaging area
sensor
light source
occluder

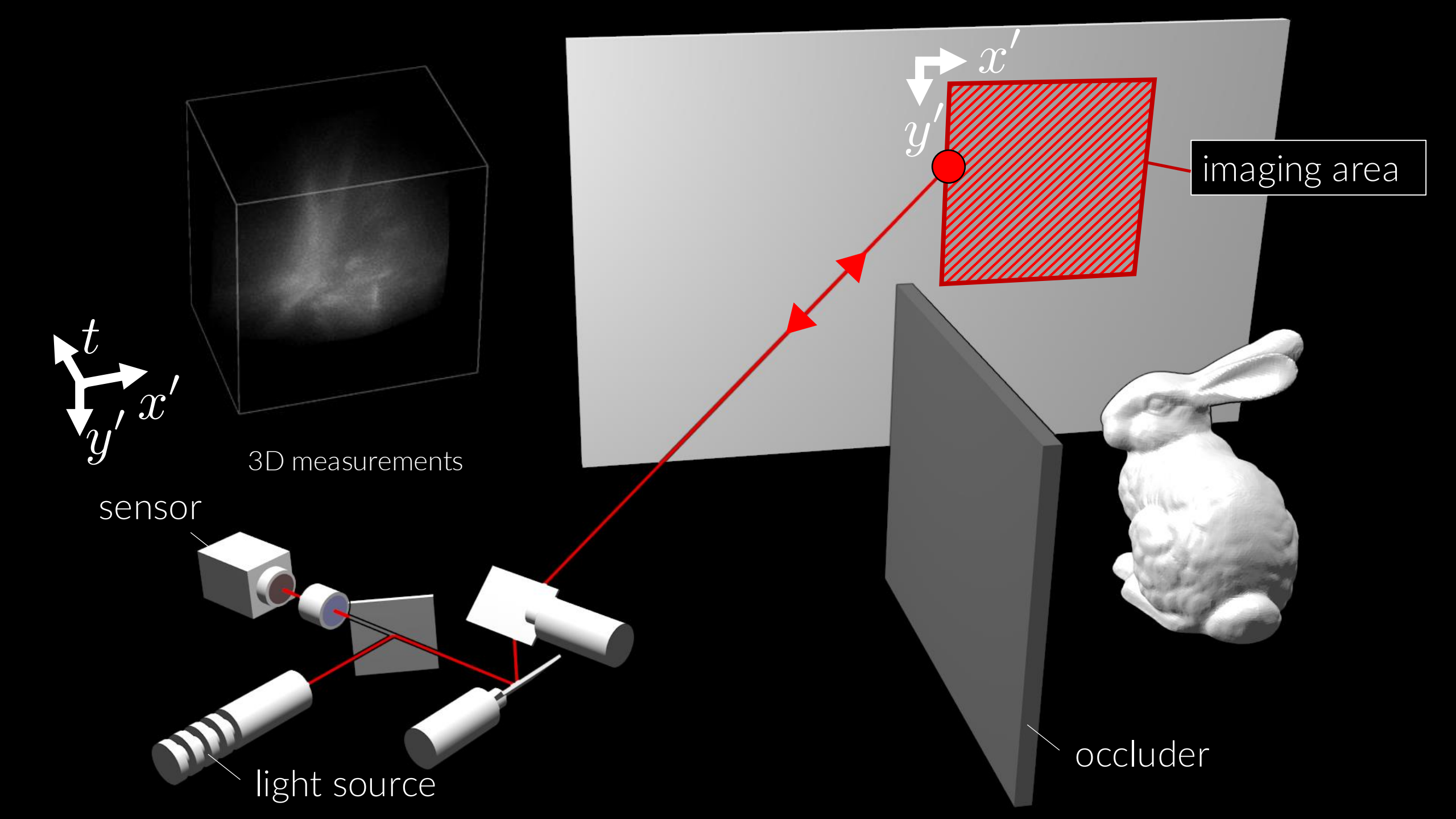
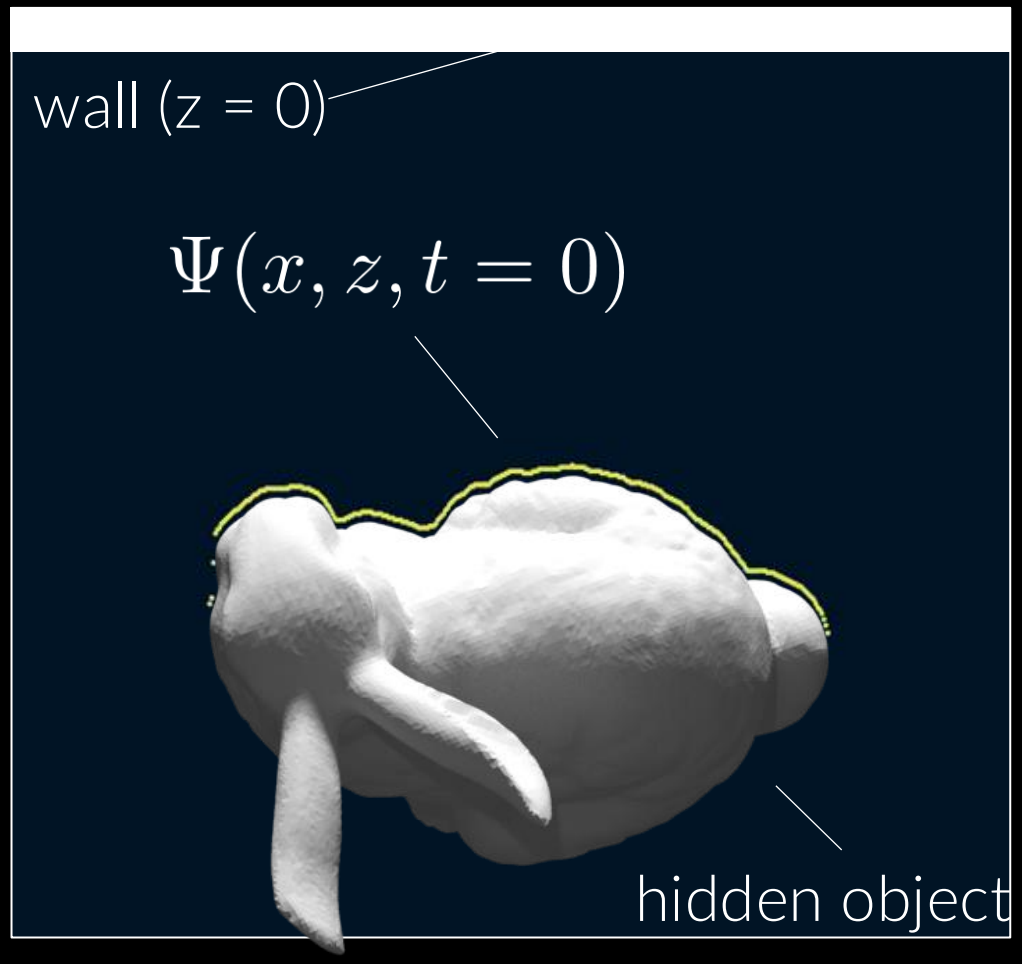

x'
y'
imaging area
t
x'
y'
3D measurements
sensor
light source
occluder

image formation model

$\Psi(x, z, t)$

wavefield



$\Psi(x, z = 0, t)$

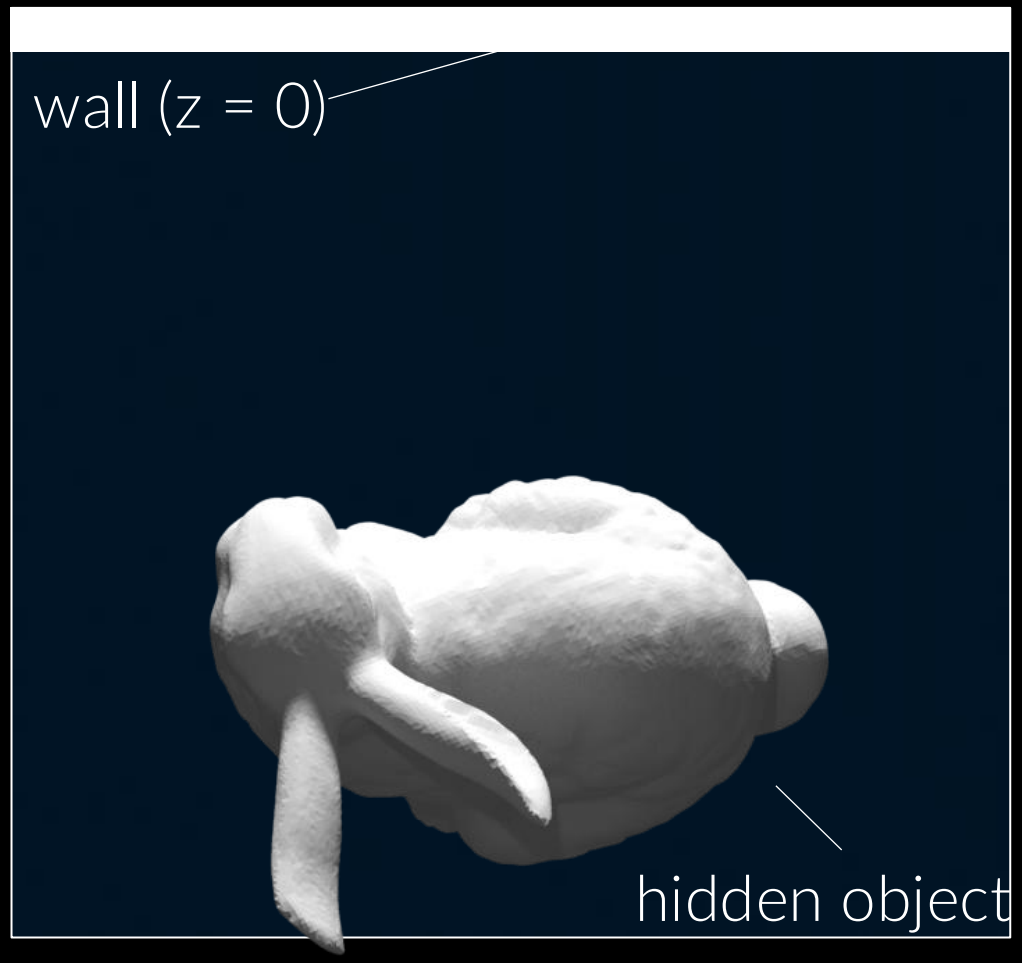
confocal measurements



general solution (time reversal)

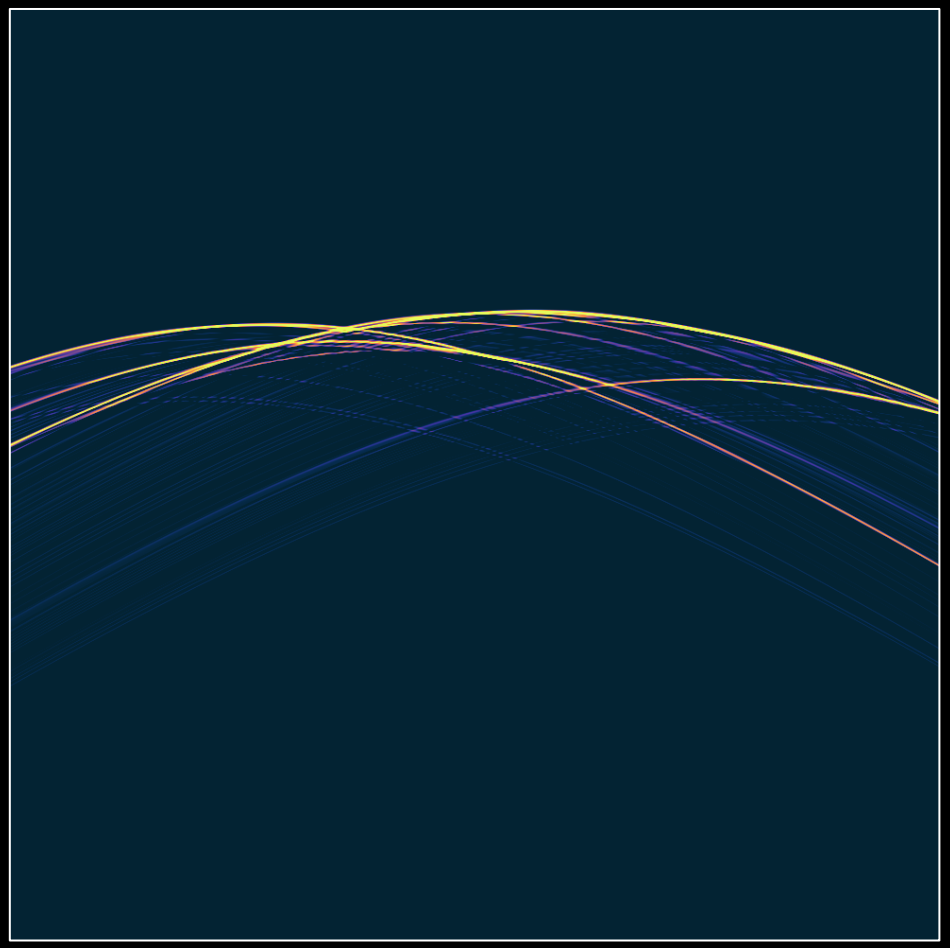
$\Psi(x, z, t = \mathbf{03.000\ ns})$

wavefield



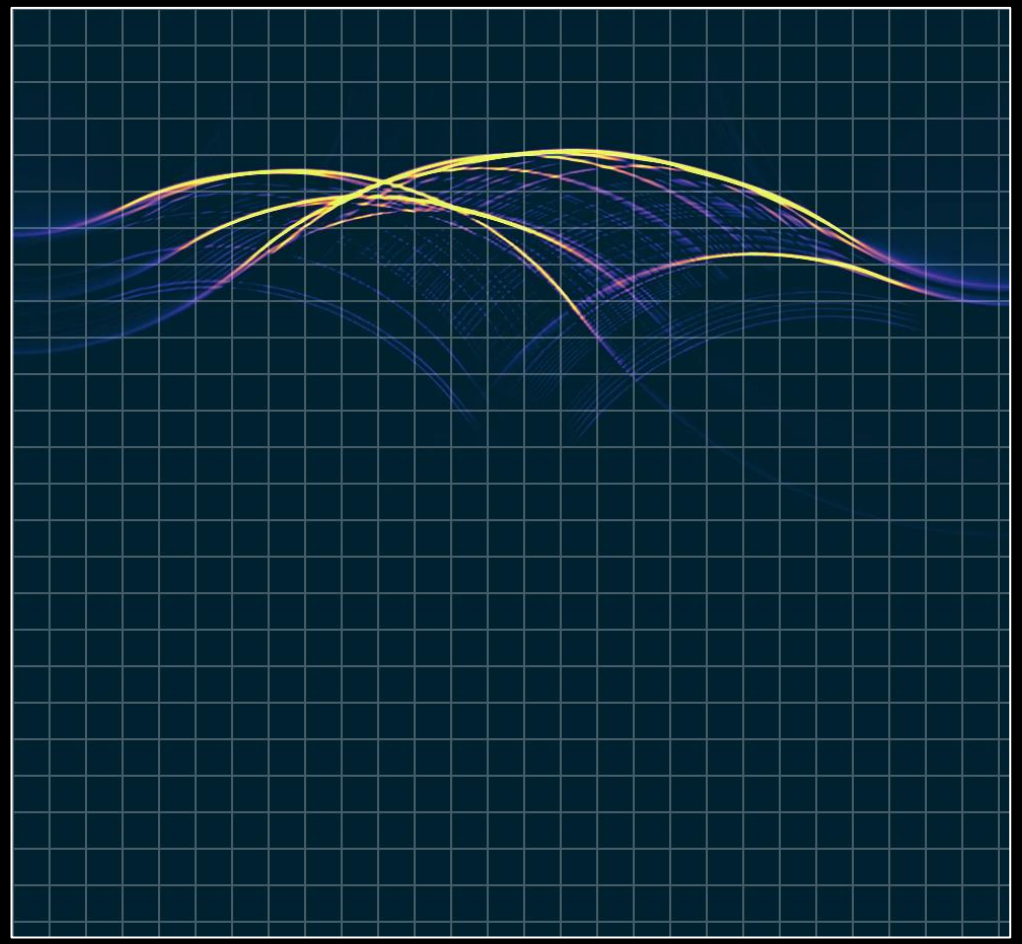
$\Psi(x, z = 0, t)$

confocal measurements



general solution (time reversal)

finite-difference
time-domain method



1. approximate wave equation with finite differences

$$\frac{\partial^2 \Psi}{\partial t^2} \approx \frac{\Psi_i^{n+1} - 2\Psi_i^n + \Psi_i^{n-1}}{(\Delta t)^2}$$

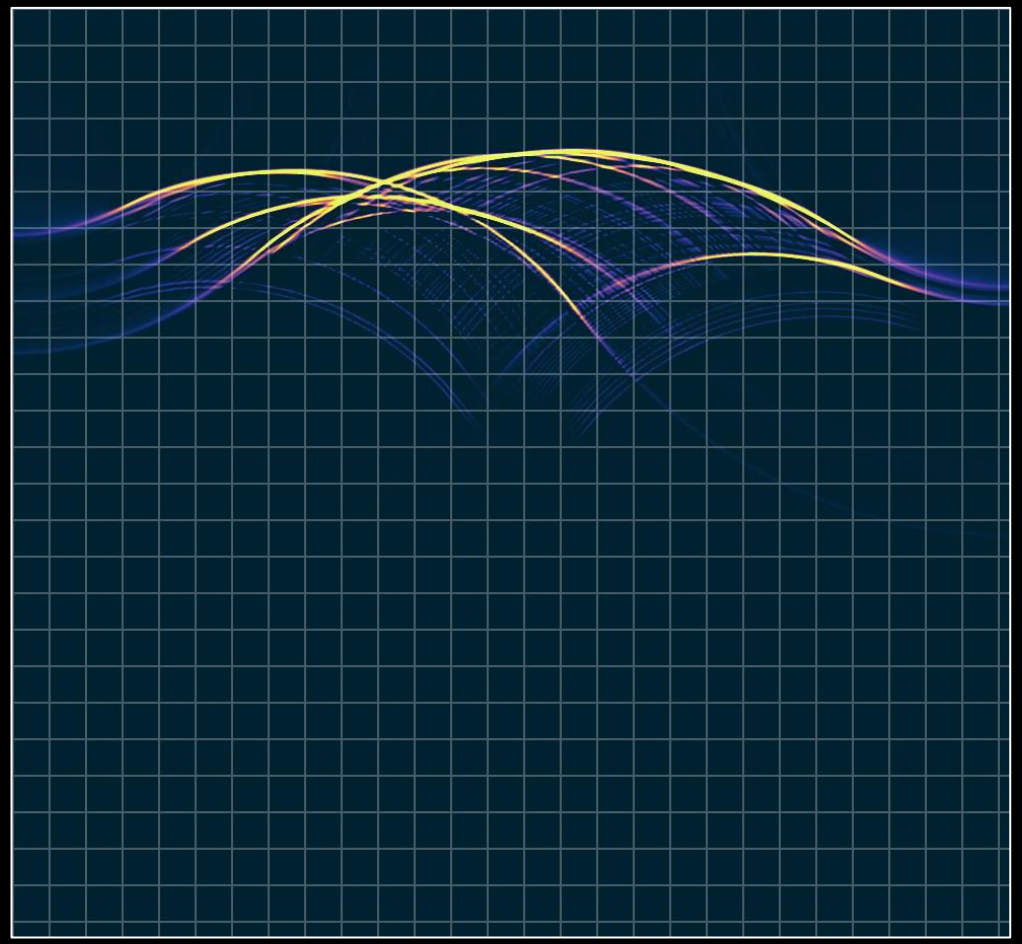
2. solve for previous timestep

$$\Psi_i^{n-1} = f\left(\Psi^n, \Psi^{n+1}\right)$$

3. repeatedly update Ψ at all grid cells

general solution (time reversal)

finite-difference
time-domain method



1. approximate wave equation with finite differences

$$\frac{\partial^2 \Psi}{\partial t^2} \approx \frac{\Psi_i^{n+1} - \quad n-1}{}$$

Slow to get t=0 at high-resolution!

2. solve ous timestep

$$\Psi_i^{n-1} = f\left(\Psi^n, \Psi^{n+1}\right)$$

3. repeatedly update Ψ at all grid cells

Frequency–wavenumber (*f*–*k*) Migration

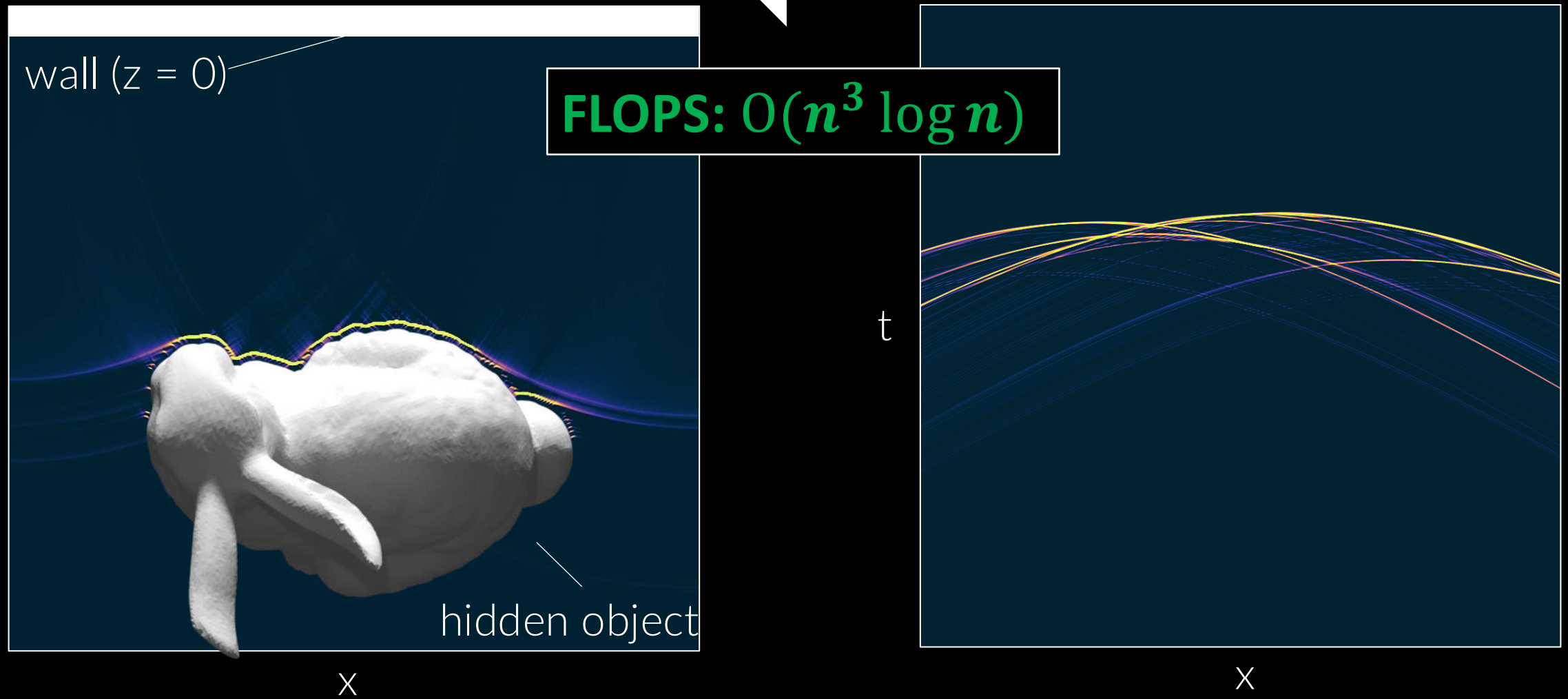
frequency–wavenumber (*f*–*k*) Migration

$\Psi(x, z, t = 0)$

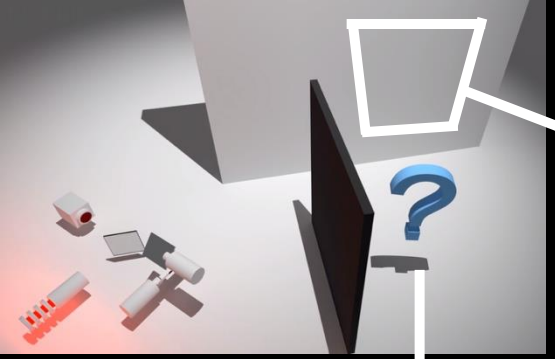
wavefield

$\Psi(x, z = 0, t)$

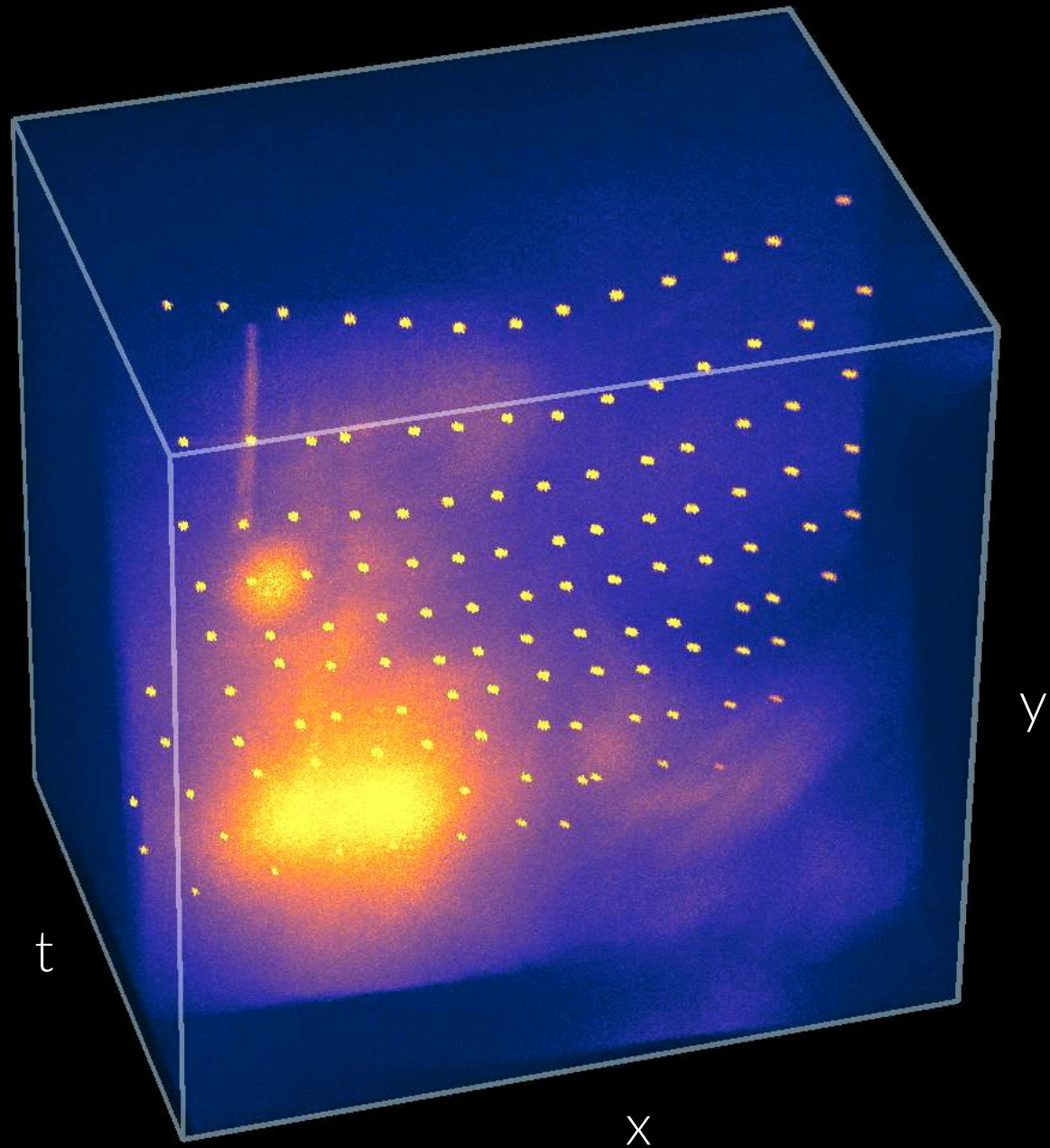
confocal measurements



FLOPS: $O(n^3 \log n)$

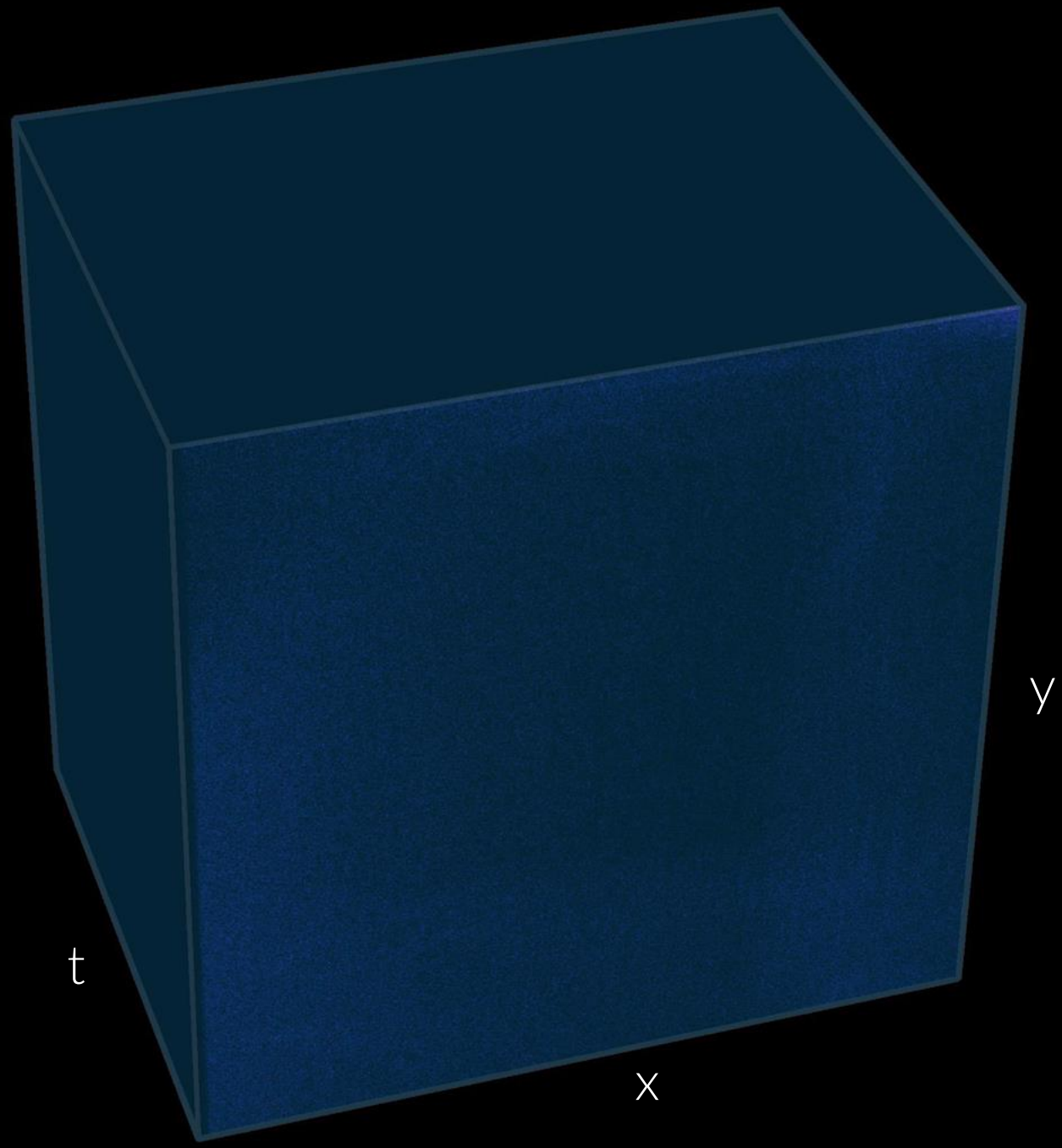






Captured Measurements

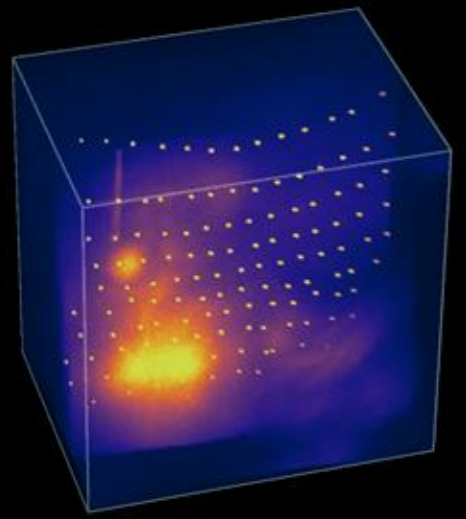



y
t
x

f-k Migration

$\Psi(x, y, t)$

Measurements (z=0)



$\bar{\Phi}(k_x, k_y, f)$

Spectrum

$\mathcal{F}$

$\Phi(k_x, k_y, k_z)$

Interpolated Spectrum

Resample

$\mathcal{F}^{-1}$

$\Psi(x, y, z)$

Hidden Volume

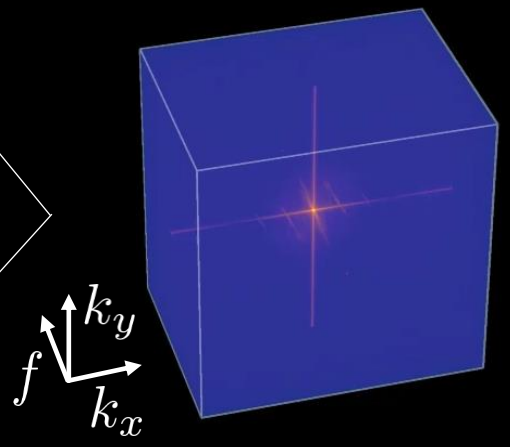
f-k Migration

Express wavefield as function of measurement spectrum (plane wave decomposition)

$$\Psi(x,y,z,t) = \iiint \bar{\Phi}(k_x,k_y,f)e^{2\pi i(k_x x+k_y y+k_z z-ft)}dk_x dk_y df$$

wavefield

Fourier transform of measurements



Set t=0 to get migrated solution

$$\Psi(x,y,z,t=0) = \iiint \bar{\Phi}(k_x,k_y,f)e^{2\pi i(k_x x+k_y y+\underbrace{k_z z}) }dk_x dk_y \underbrace{df}$$

Almost an inverse Fourier Transform!

f-k Migration

Set t=0 to get migrated solution

$$\Psi(x, y, z, t = 0) = \iiint \bar{\Phi}(k_x, k_y, f) e^{2\pi i(k_x x + k_y y + \underbrace{k_z z}) dk_x dk_y \underbrace{df}}$$

Almost an inverse Fourier Transform!

Use dispersion relation[1] to perform substitution of variables

$$f = v\sqrt{k_x^2 + k_y^2 + k_z^2}$$

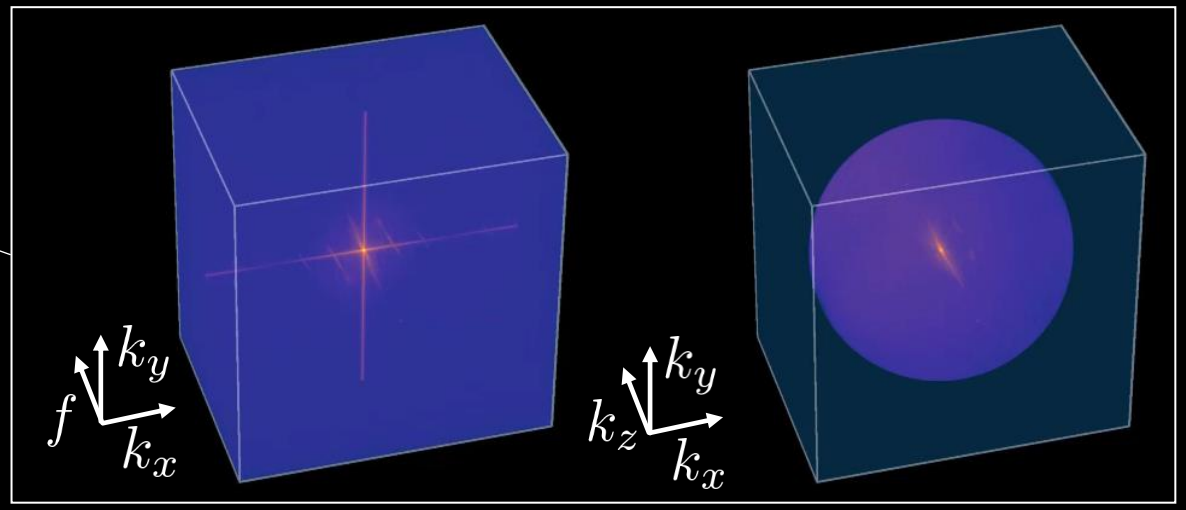
$$\boxed{f \Rightarrow k_z}$$

[1]Georgi, Howard. *The physics of waves*. Englewood Cliffs, NJ: Prentice Hall, 1993.

Use dispersion relation[1] to perform substitution of variables

$$f = v\sqrt{k_x^2 + k_y^2 + k_z^2}$$

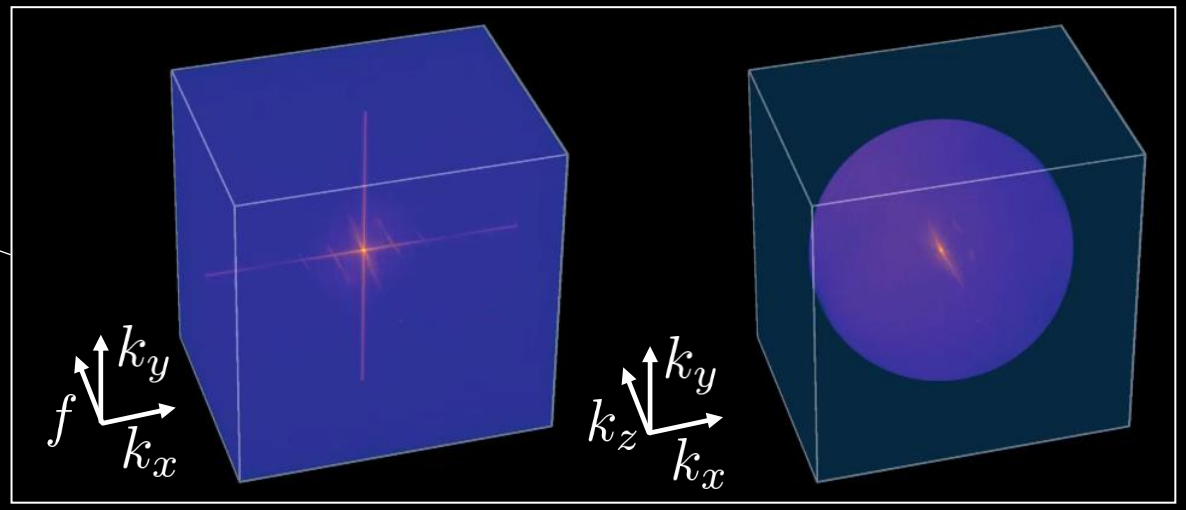
$$\boxed{f \Rightarrow k_z}$$

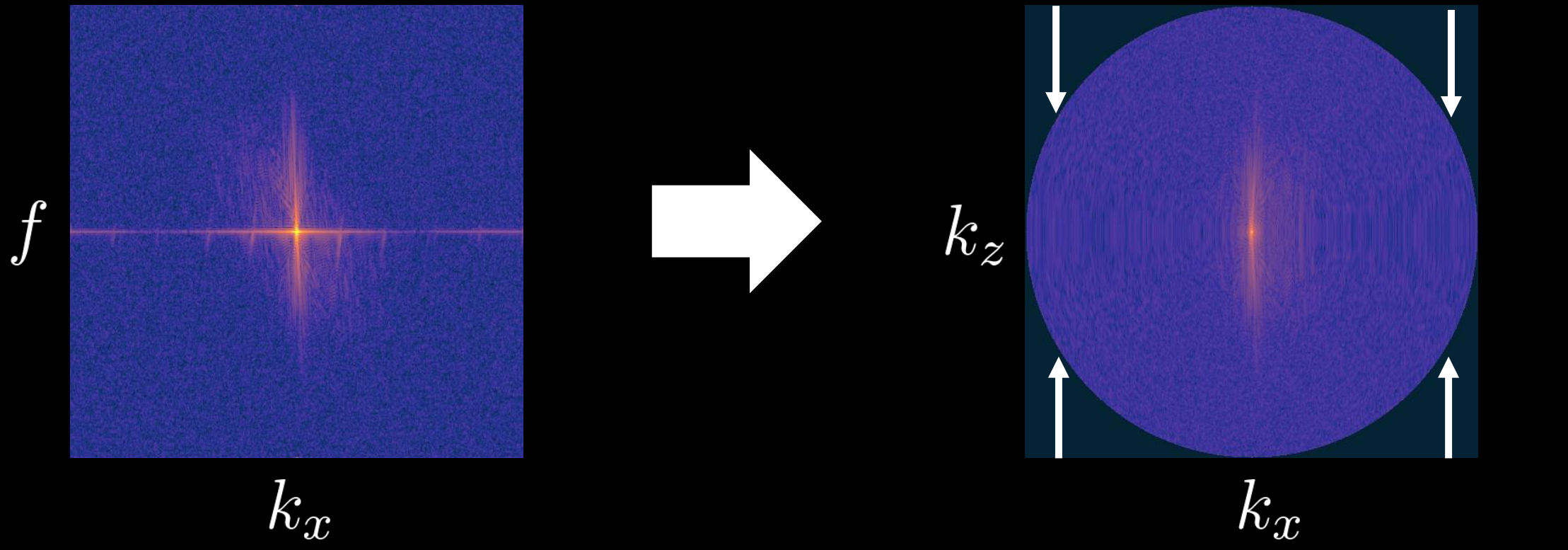


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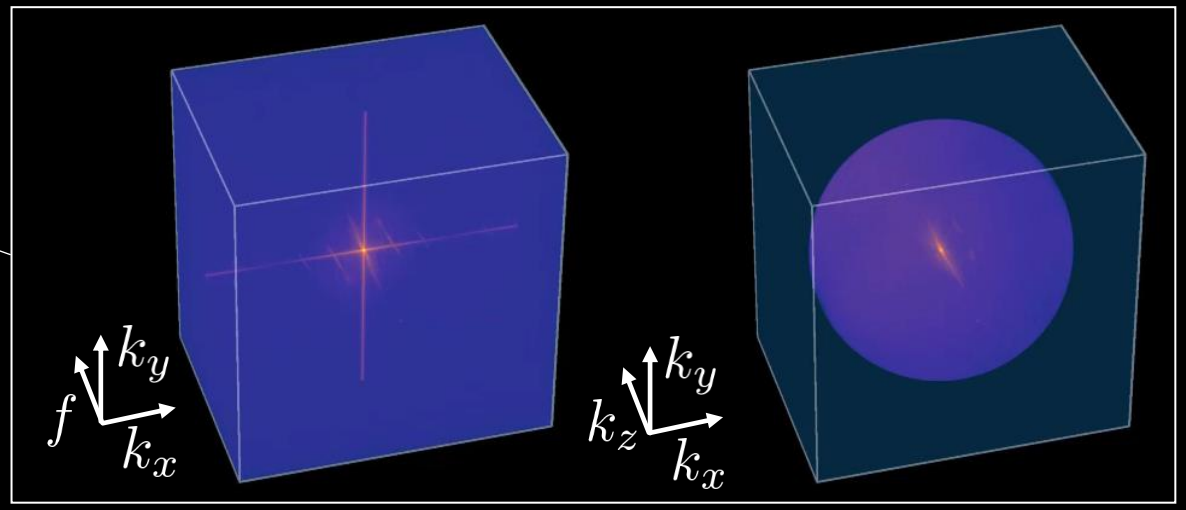




Use dispersion relation[1] to perform substitution of variables

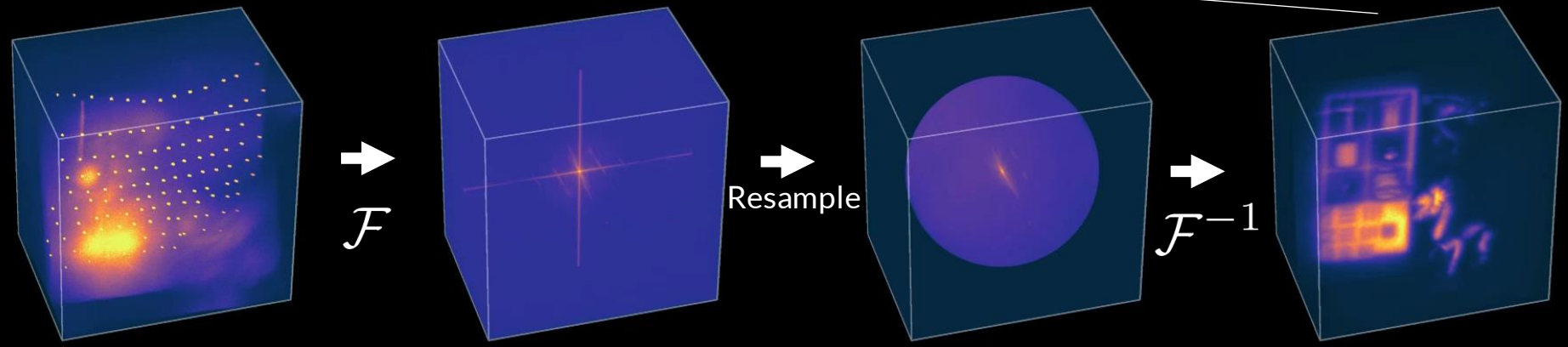
$$f = v\sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$\boxed{f \Rightarrow k_z}$$



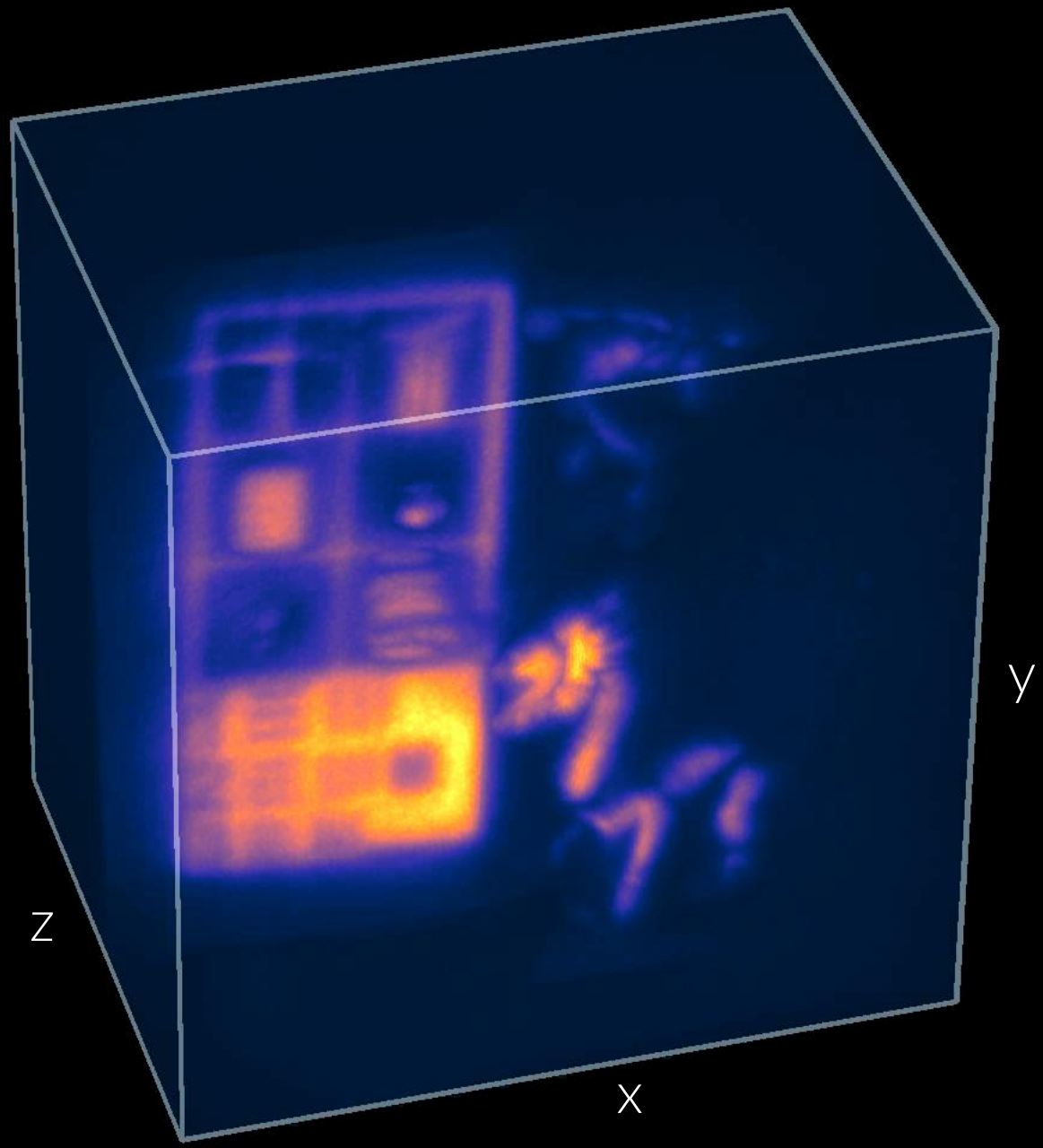
The migrated solution is an inverse Fourier Transform!

$$\Psi(x, y, z, t = 0) = \iiint \Phi(k_x, k_y, k_z) e^{2\pi i(k_x x + k_y y + k_z z)} dk_x dk_y dk_z$$



f-k Migration



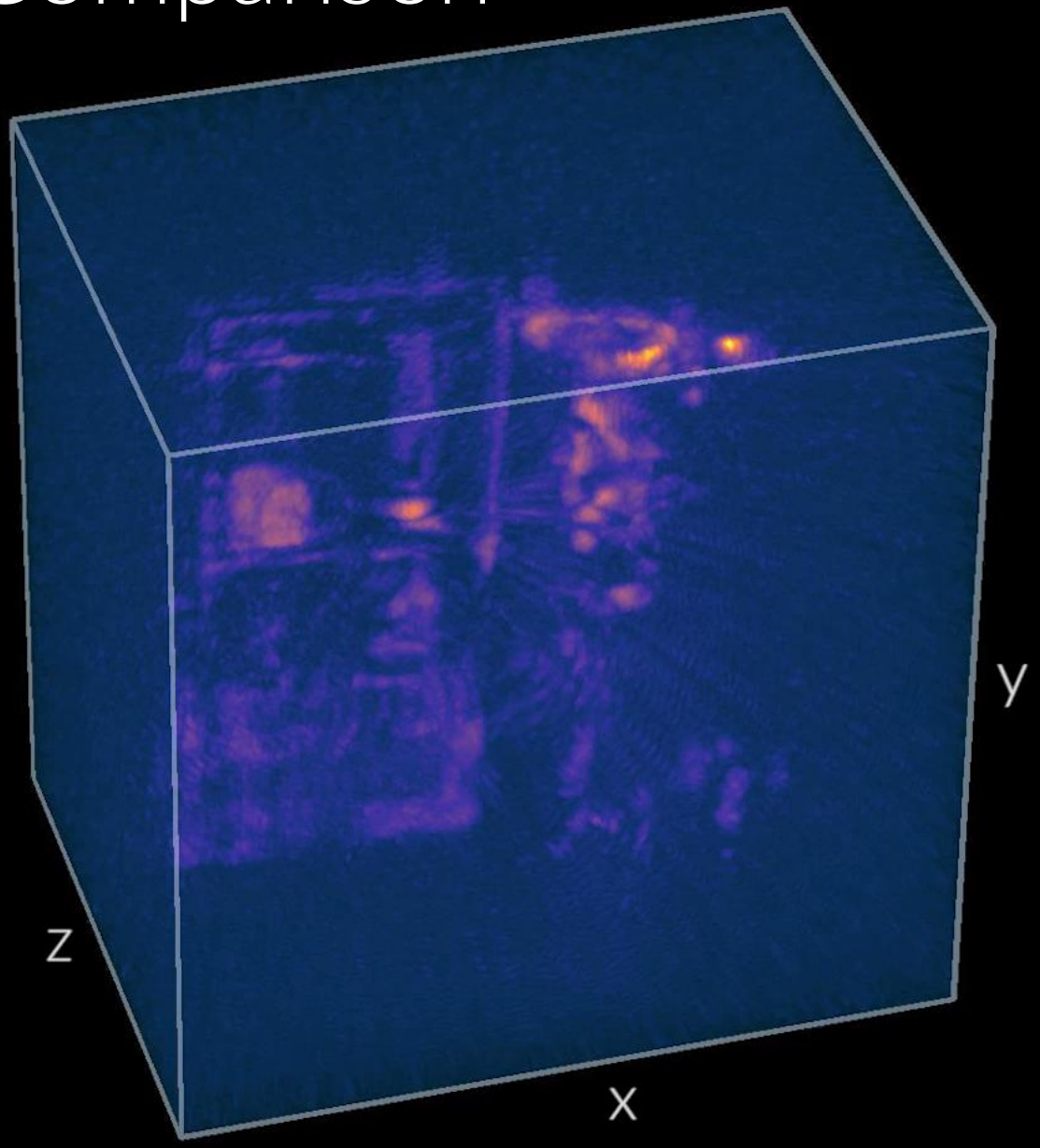


Dimensions: 2 x 2 m
Exposure: 180 min
Time Resolution: 32 ps

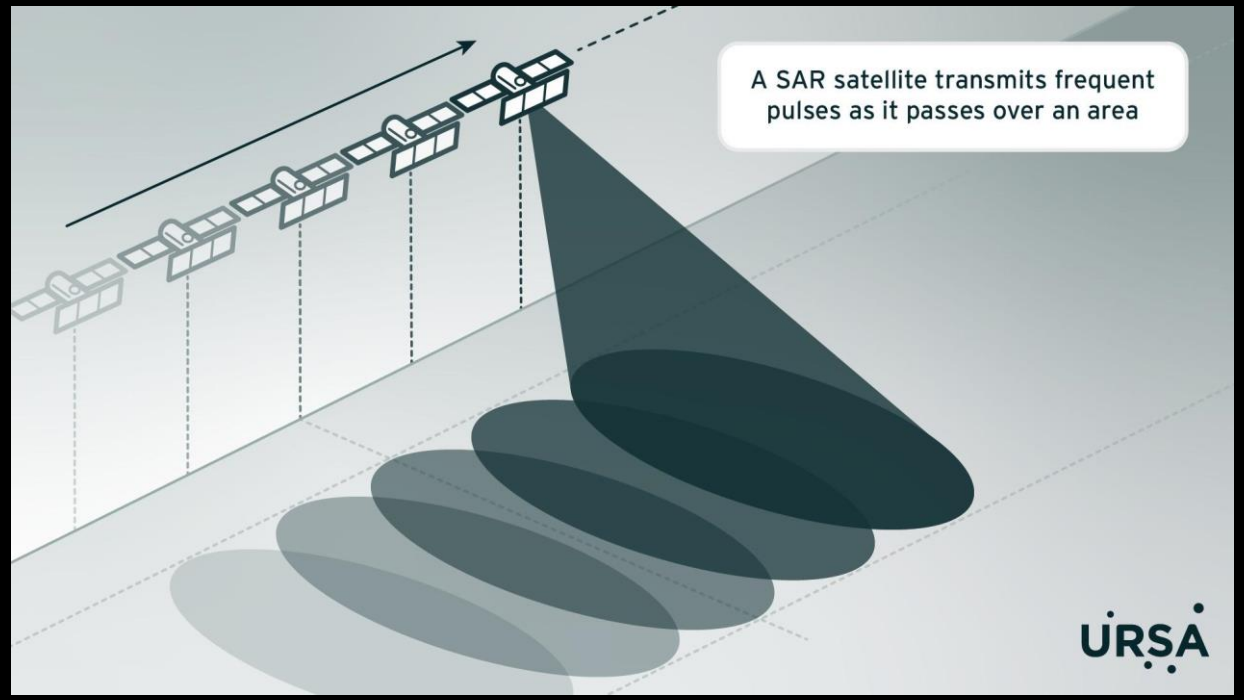
Reconstruction Comparison

dimensions: 2 m x 2 m x 1.5 m





Filtered Backprojection





Key message:
SAR turns motion into resolution—computationally synthesizing an antenna much larger than the physical one.