# Announcements

- Slides & resources are on the course website: https://radarml.github.io/18848/
- Lab 1 will be released Tuesday, 09/08 (Due 09/29)
  - Demo on Tuesday!
  - All materials will be posted on the lab website
- Lab kits available for checkout starting Tuesday
  - Max 1 week per student
  - 3 kits reserved for usage in the lab (hours: 9-5 M-F)
  - Auditors are welcome to do the lab, but cannot check out lab kit

# Radar Signal Processing

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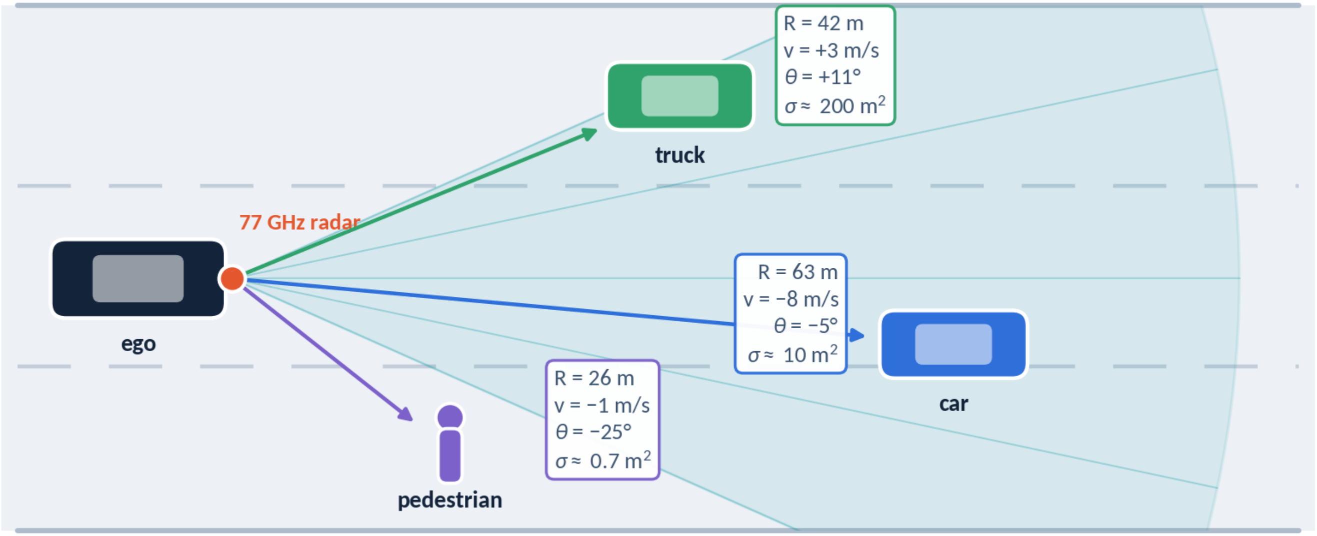
# Overview: Extracting Target Information from Radar Signals

A wireless channel describes how the signal goes from the transmitter to the receiver.

Sensing system / radar relies on channel to acquire multipath and object information.

**Range** **Velocity** **Angle** **Reflectivity**




Does a stronger echo always mean a closer object?
Not awalys. That's why we need the radar range equation.

# Radar Range Equation

# Radar Range Equation

How much power a radar receives from a target

Antenna **G**ain

**R**adar **C**ross **S**ection

Effective **A**perture

Transmitting **P**ower

$$P_r = \frac{P_t\, G_t\, \sigma\, A_e}{(4\pi)^2\, R^4}$$

**Radar-related:** power, gain, aperture

**Target-related:** RCS, range

# Antenna Gain

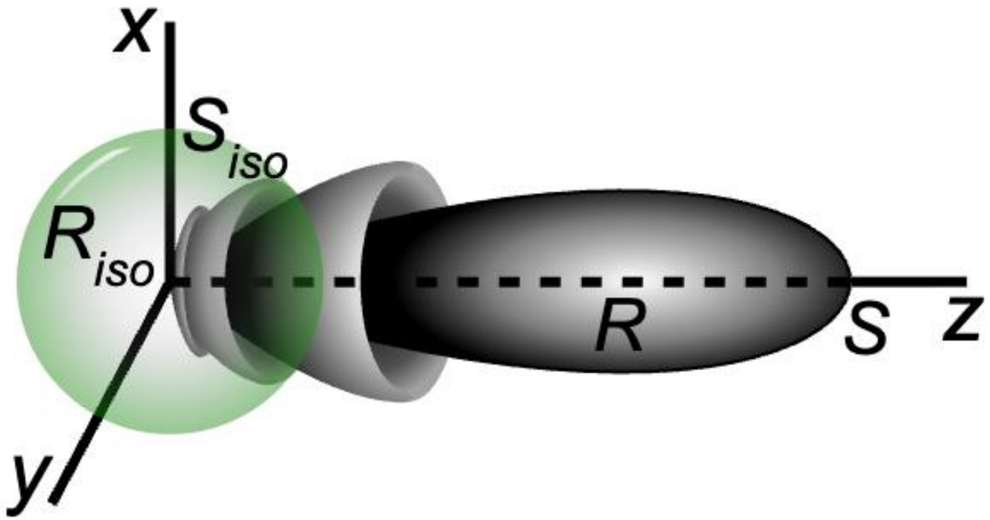
An antenna converts between electrical signals and electromagnetic waves.

Gain: How well the antenna converts input power into waves headed in one direction.

Isotropic

Directional

**Radiation Pattern**




Pros and Cons?

# Antenna

An antenna converts between electrical signals and electromagnetic waves.

| Patch Antenna | Horn Antenna | Parabolic Antenna | Phased Array Antenna |
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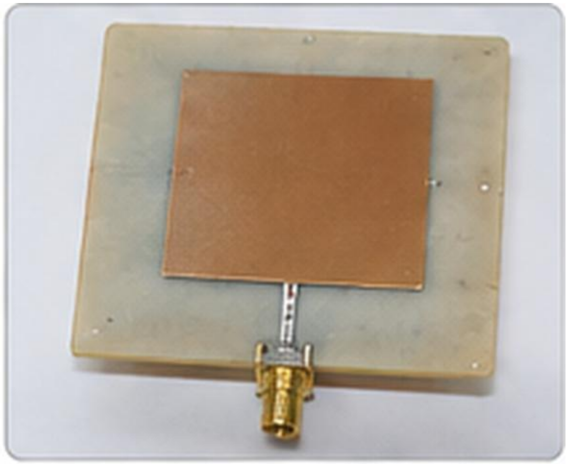
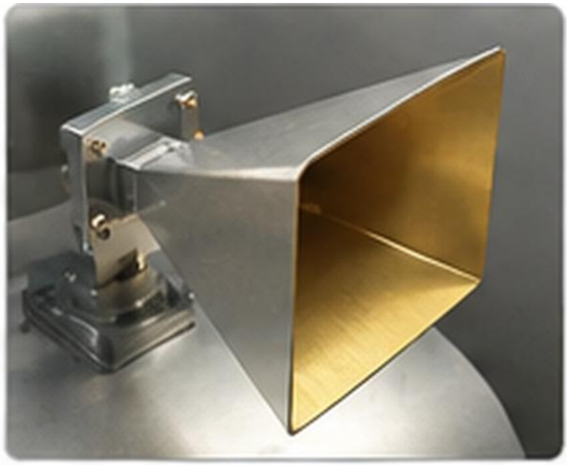
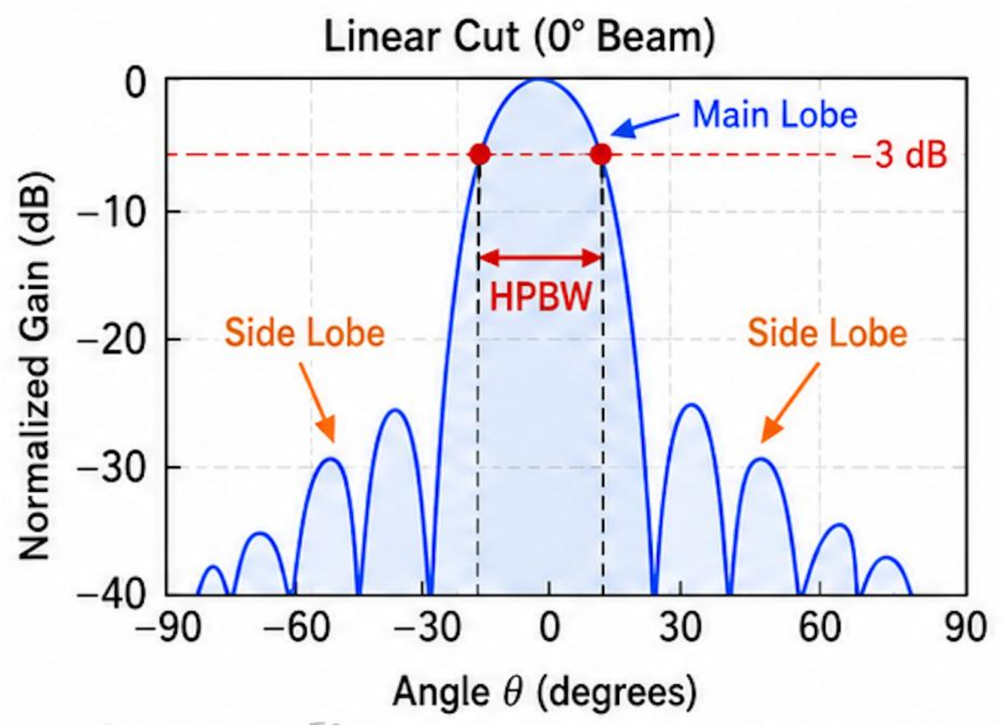
|  |  |  |  |

# Antenna Gain

## Main lobe, side lobe and Half-Power Beamwidth (HPBW)




The main lobe tells us where the antenna is looking.

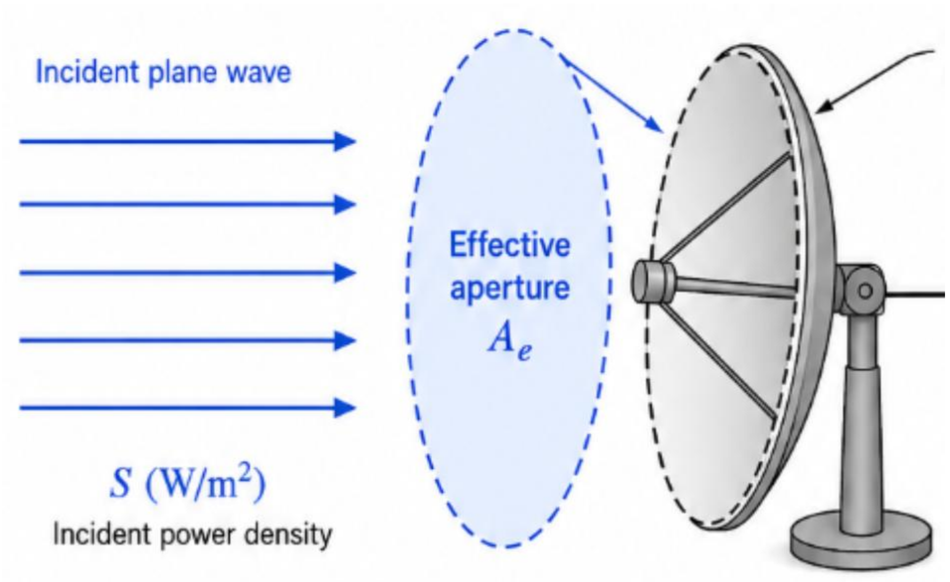
The side lobes are weaker beams pointing in other directions.

HPBW: Angular distance between the two points where the power drops to half of the peak value, or −3 dB.

**Why are there side lobes?**

# Effective Aperture

How effective an antenna is at receiving power

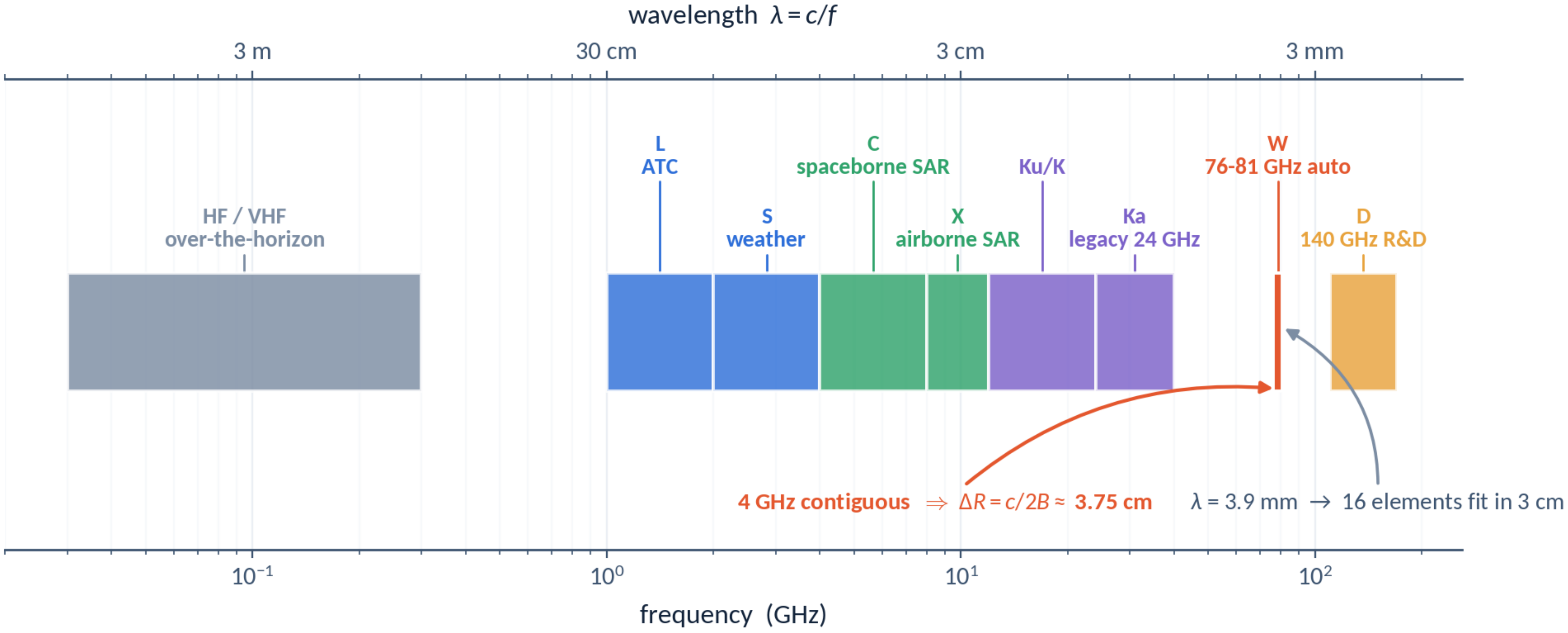



$$A_e = \frac{G_r \lambda^2}{4\pi}$$

# Wavelength

How should you choose the right frequency for your radar?

-> It depends on the range and target.



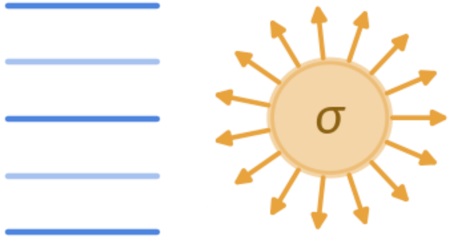

Lower frequency -> Less attenuation, more penetration

Higher frequency -> Smaller antenna, better resolution

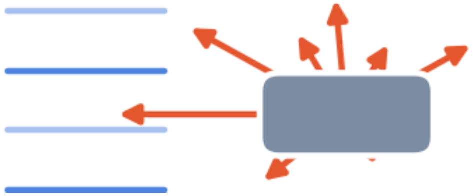
# Radar Cross Section

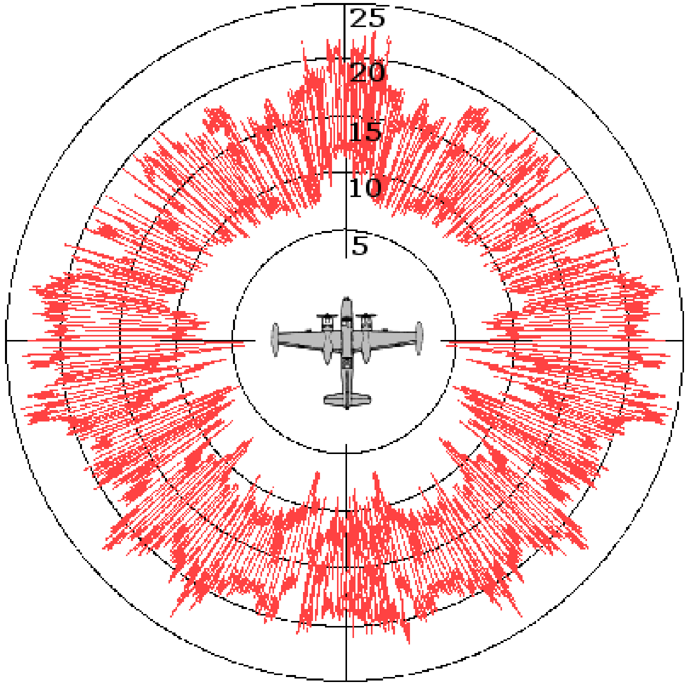
How strongly a target reflects radar energy back to the radar.

Ideal Case




Real World





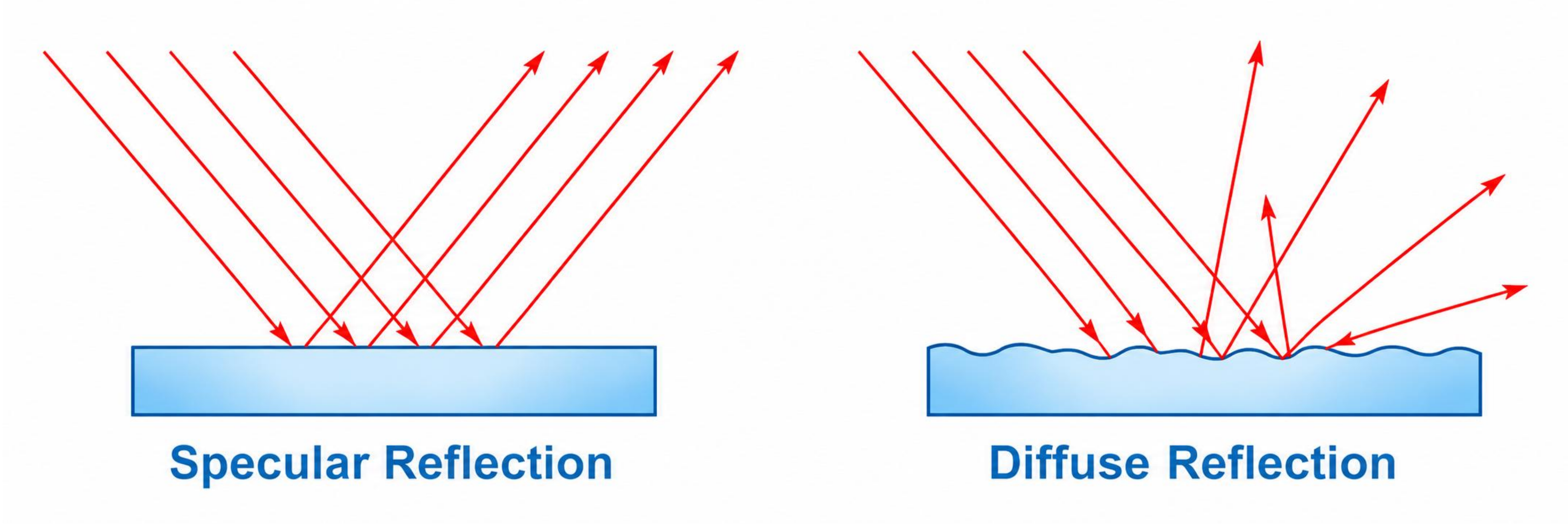

**What factors affect RCS?**

**Size, material, orientation, shape, surface roughness**

# Radar Cross Section

**Specular reflection** happens when the surface is smooth relative to the wavelength.

**Diffuse reflection** happens when the surface is rough relative to the wavelength.



# Deriving the Radar Range Equation

**1 isotropic TX**

$$S_{iso} = \frac{P_t}{4\pi R^2}$$

power spreads over a sphere of radius R

**2 focus with gain**

$$S_{inc} = \frac{P_t G_t}{4\pi R^2}$$

antenna gain $G_t$ concentrates energy into the beam

**3 target re-radiates**

$$S_{re} = \frac{P_t G_t}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2}$$

RCS $\sigma$ = equivalent area that scatters isotropically

**4 capture with aperture**

$$P_r = S_{re} A_e, \quad A_e = \frac{G_r \lambda^2}{4\pi}$$

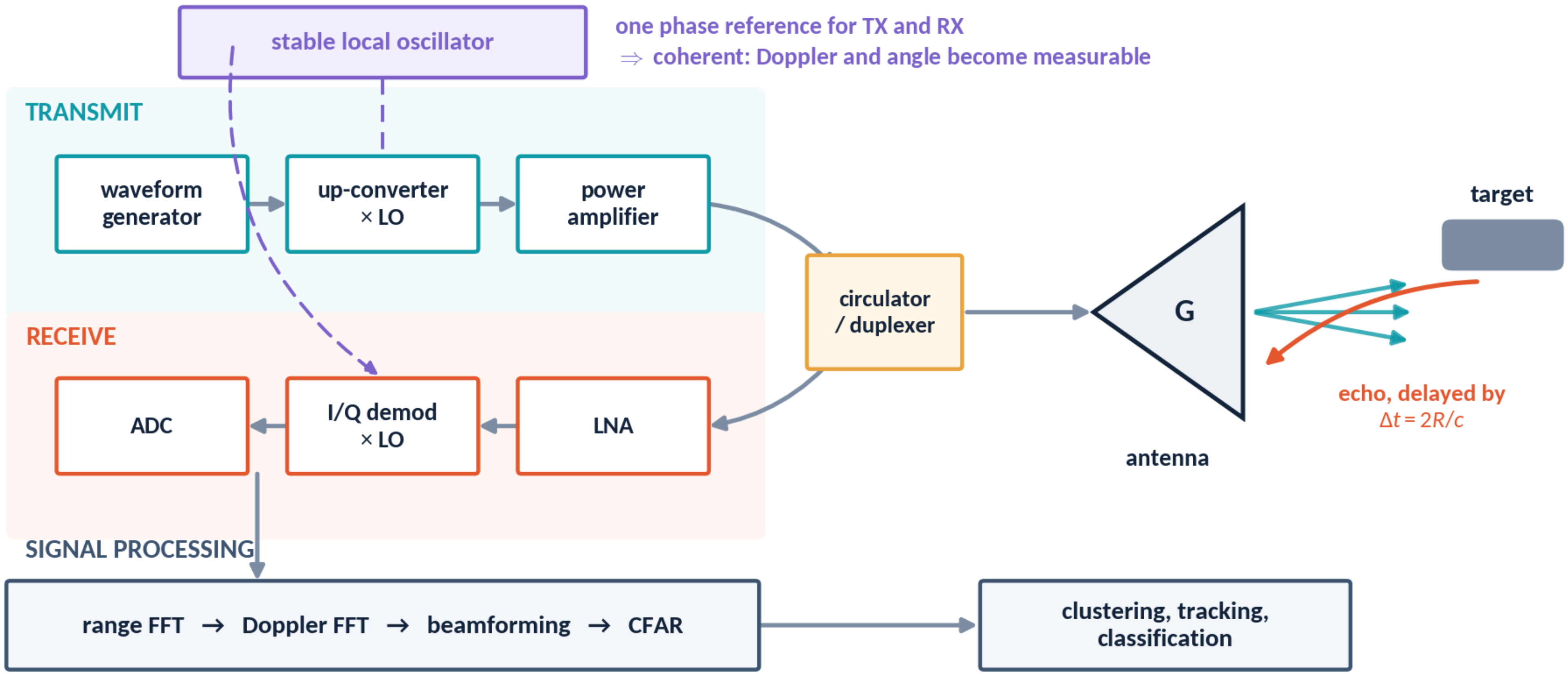
effective aperture of the receive antenna

**Monostatic radar range equation**

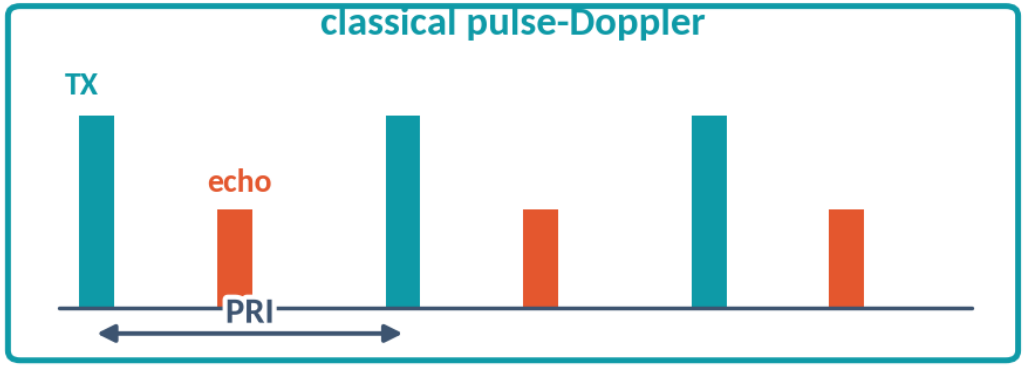
$$P_r = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4 L} \qquad \text{with } G_t = G_r = G: \qquad P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L}$$

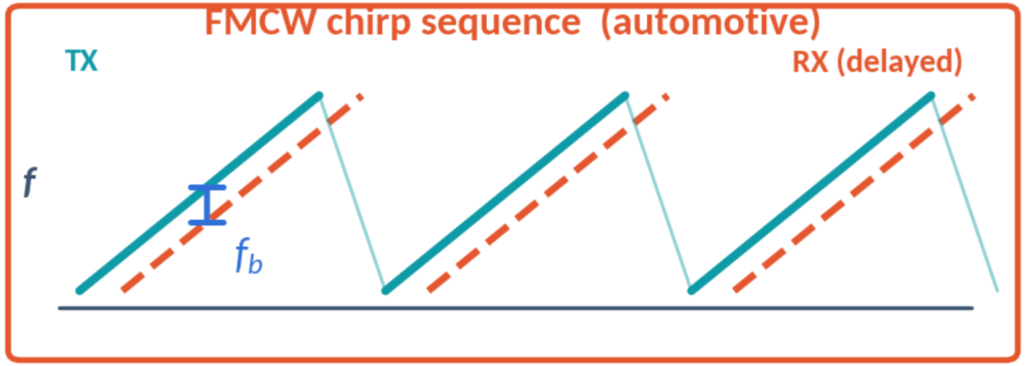
# Pulse Radar System

# Radar System



# Pulse Radar and Continuous-Wave Radar




## Pros and Cons?

**Pulse radar:**

**Pros:**

High peak transmit power gives good long-range detection.
Tx and Rx can be separated in time, reducing self-interference.

**Cons:** blind range and range ambiguity

**CW radar:**

**Pros:**

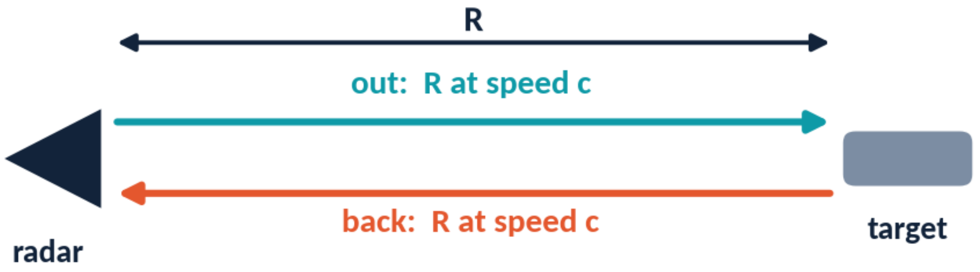
No pulse-related blind range
Continuous observation gives high Doppler sensitivity.

**Cons:** Cannot directly measure range because there is no timing reference;
Strong Tx leakage into Rx is a major challenge.

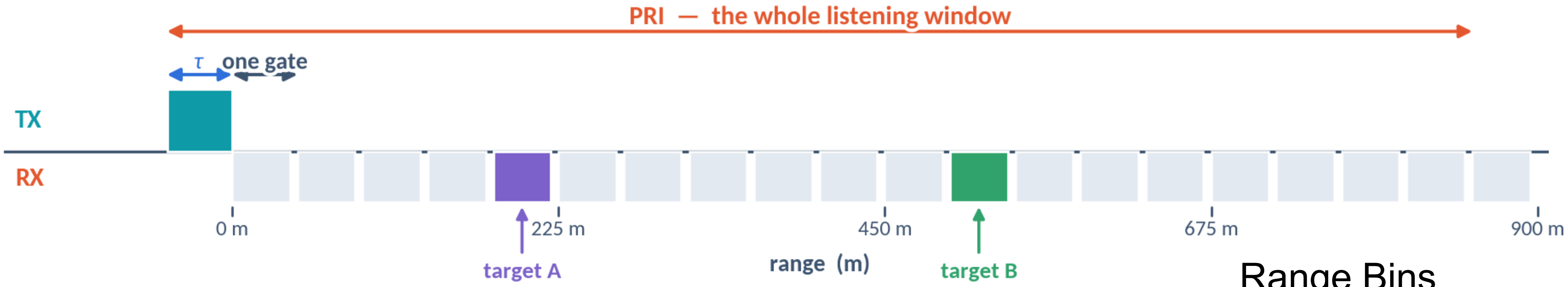
# Range Estimation

# Range Estimation

The time delay of the received pulse encodes the target's range.

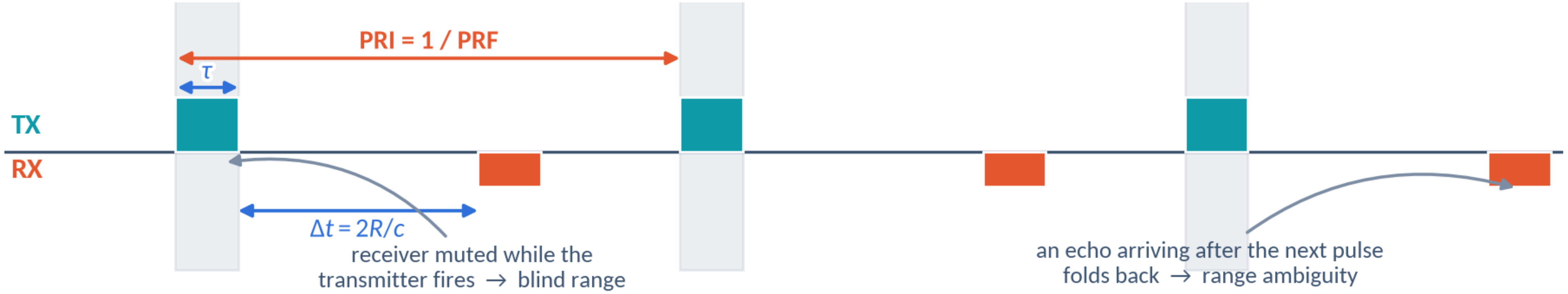



$$R = \frac{c\,\Delta t}{2}$$




Range Bins

# Blind Range and Range Ambiguity

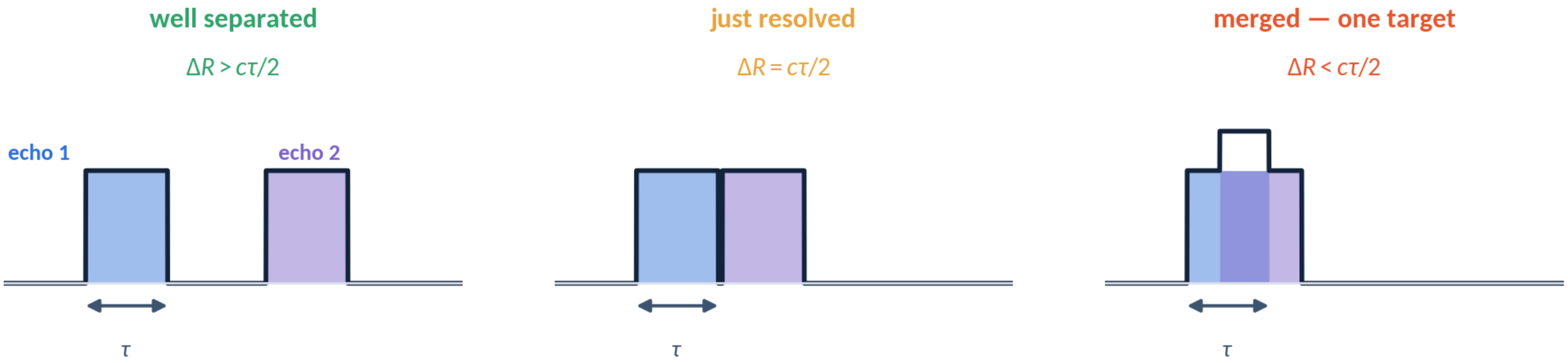



How can we deal with blind range and range ambiguity?

Blind range: shorter pulses, seperate Tx/Rx, FMCW

Range ambiguity: lower PRF, multiple PRFs

# Range Resolution




How can we manage the range-resolution trade-off?
Pulse compression (next lecture)

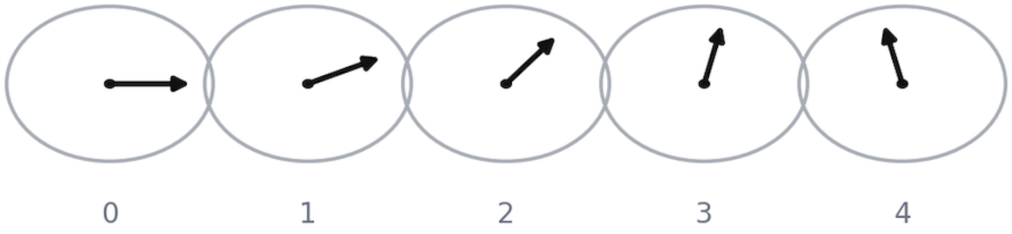
# Doppler Estimation

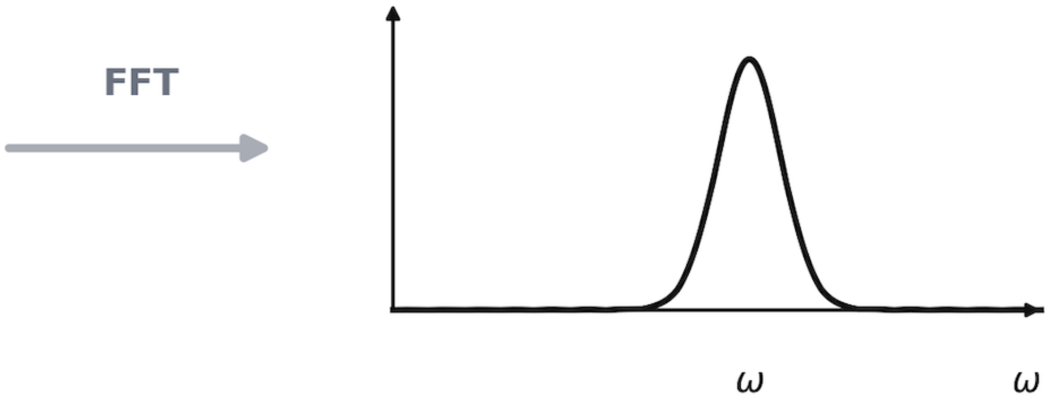
Slides credit: Texas Instruments

# FFT on a complex sequence

A constant phase increment across samples becomes a spectral peak.

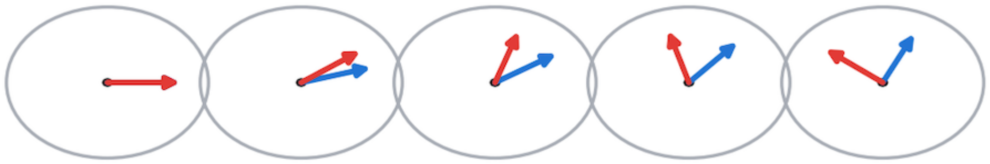
## One rotating phasor

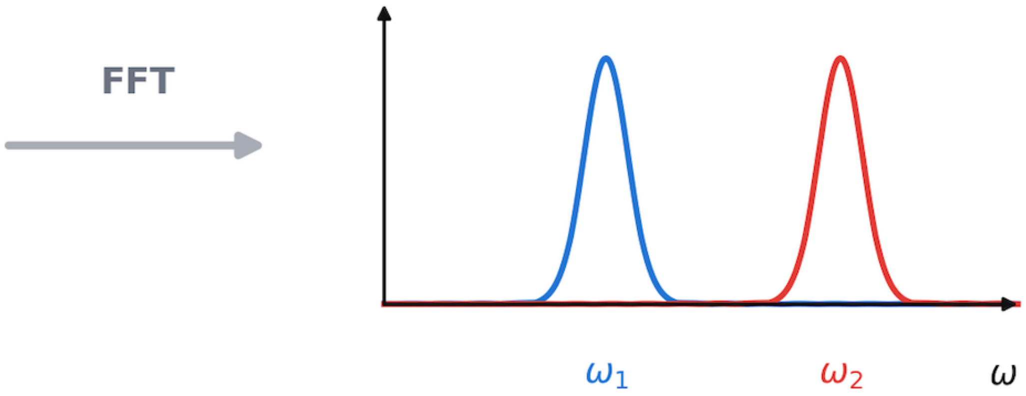



$$x[k] = Ae^{jk\omega}$$

## Sum of two rotating phasors



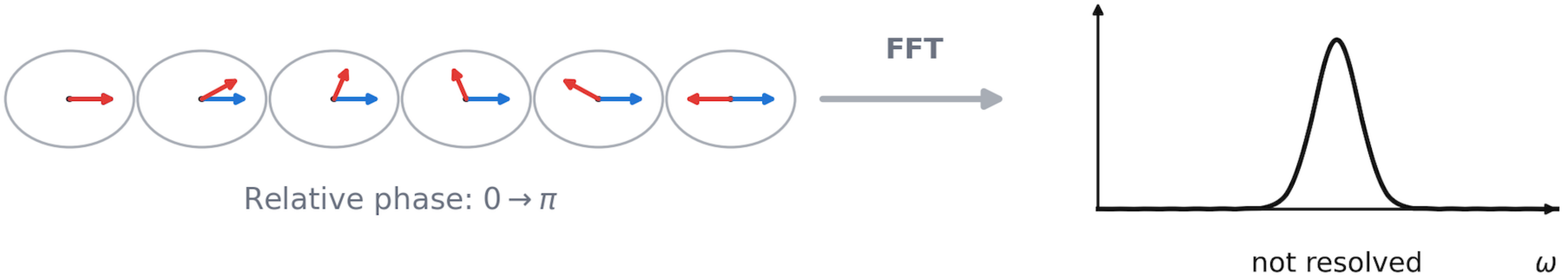



$$x[k] = A_1 e^{jk\omega_1} + A_2 e^{jk\omega_2}$$

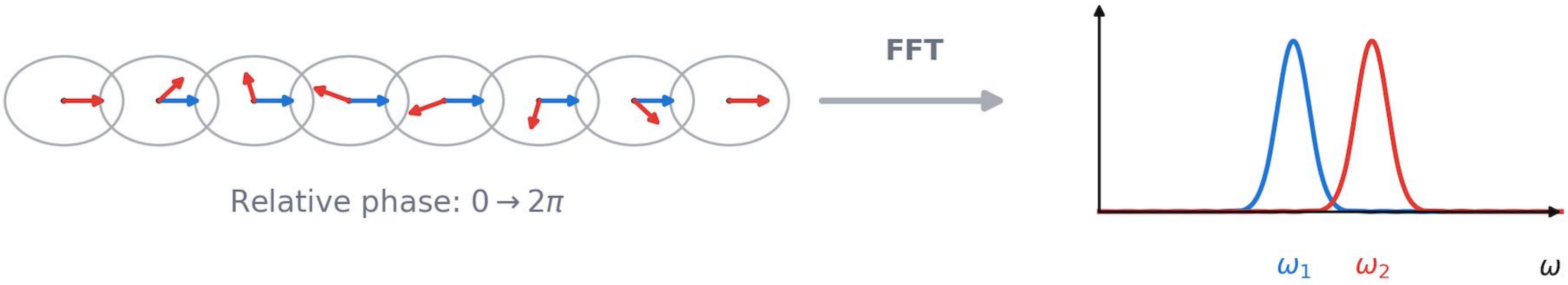
# FFT Resolution

When w1 = 0 and w2 = pi/N, the relative phase grows by only pi over N.

N samples




2N Samples




**The longer the sequence,**
**The better the resolution**

$$\Delta f = \frac{1}{T}$$

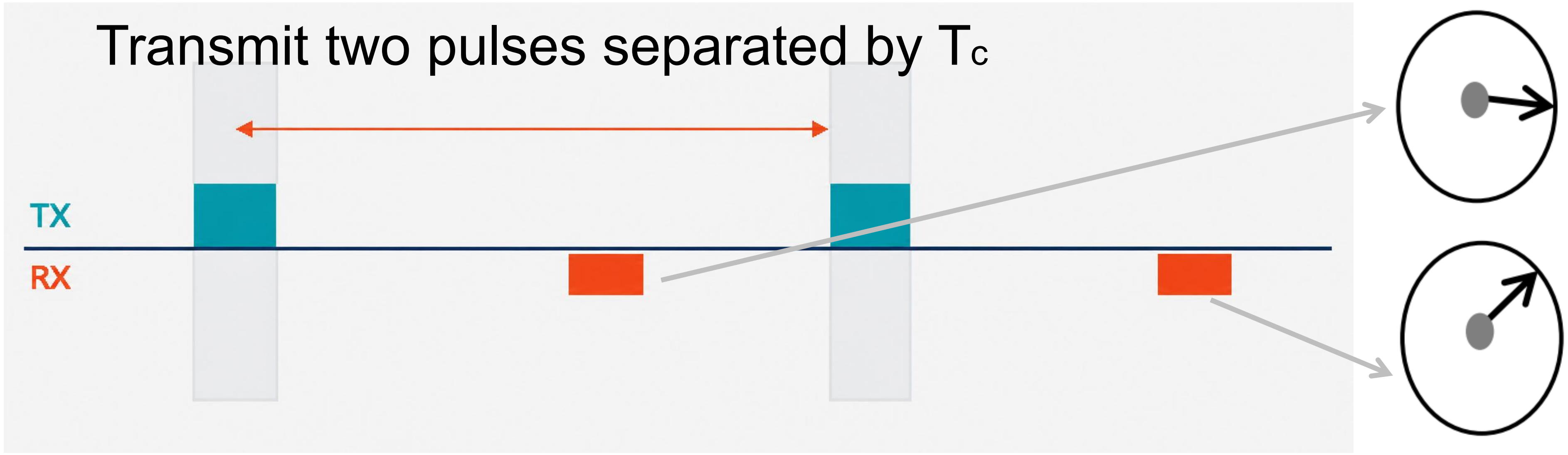
cycles / second

$$\Delta \omega = \frac{2\pi}{N}$$

$= \frac{1}{N}$ cycles/sample

# Velocity Estimation using Two Pulses

Phase changes across pulses encode the targets' velocity.

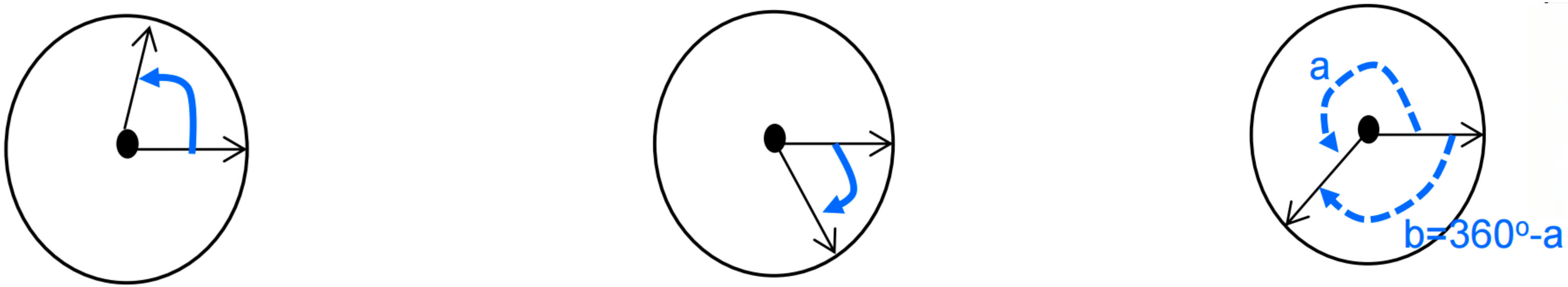



The phase difference corresponds to a target displacement of $v_r T_c$

$$\Delta\phi = \frac{4\pi v_r T_c}{\lambda}, \quad v_r = \frac{\lambda \Delta\phi}{4\pi T_c}$$

# Maximum Unambiguous Velocity

Move away from the radar | Move towards the radar | Which direction?

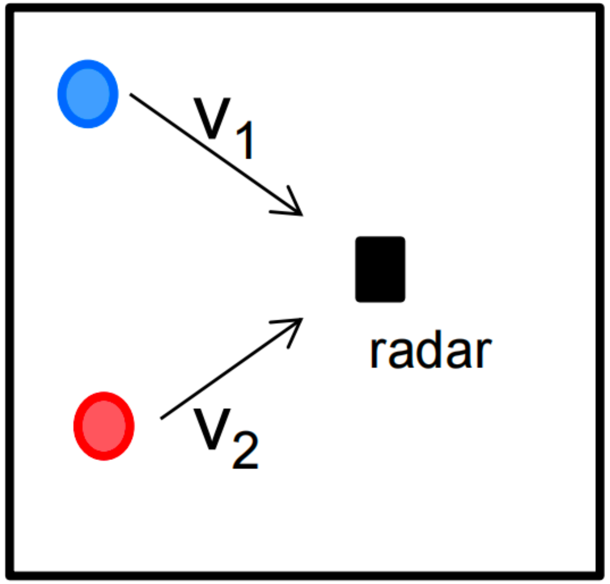


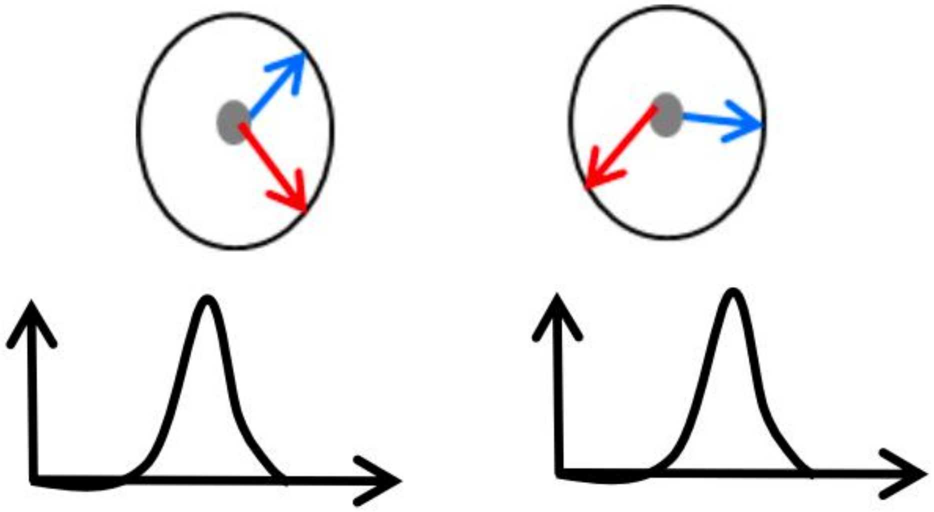

The measurement is unambiguous only if | ω | < π.

$$|\omega| < \pi$$

$$\left|\frac{4\pi v T_c}{\lambda}\right| < \pi \Rightarrow |v| < \frac{\lambda}{4T_c}$$

# Measuring Velocity with Multiple Objects



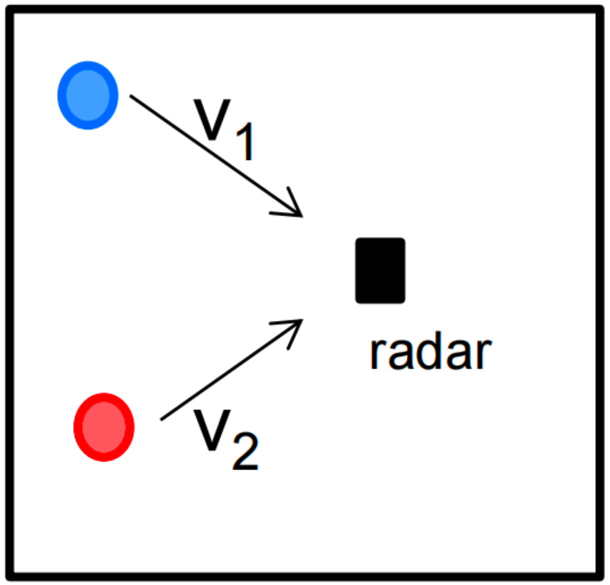



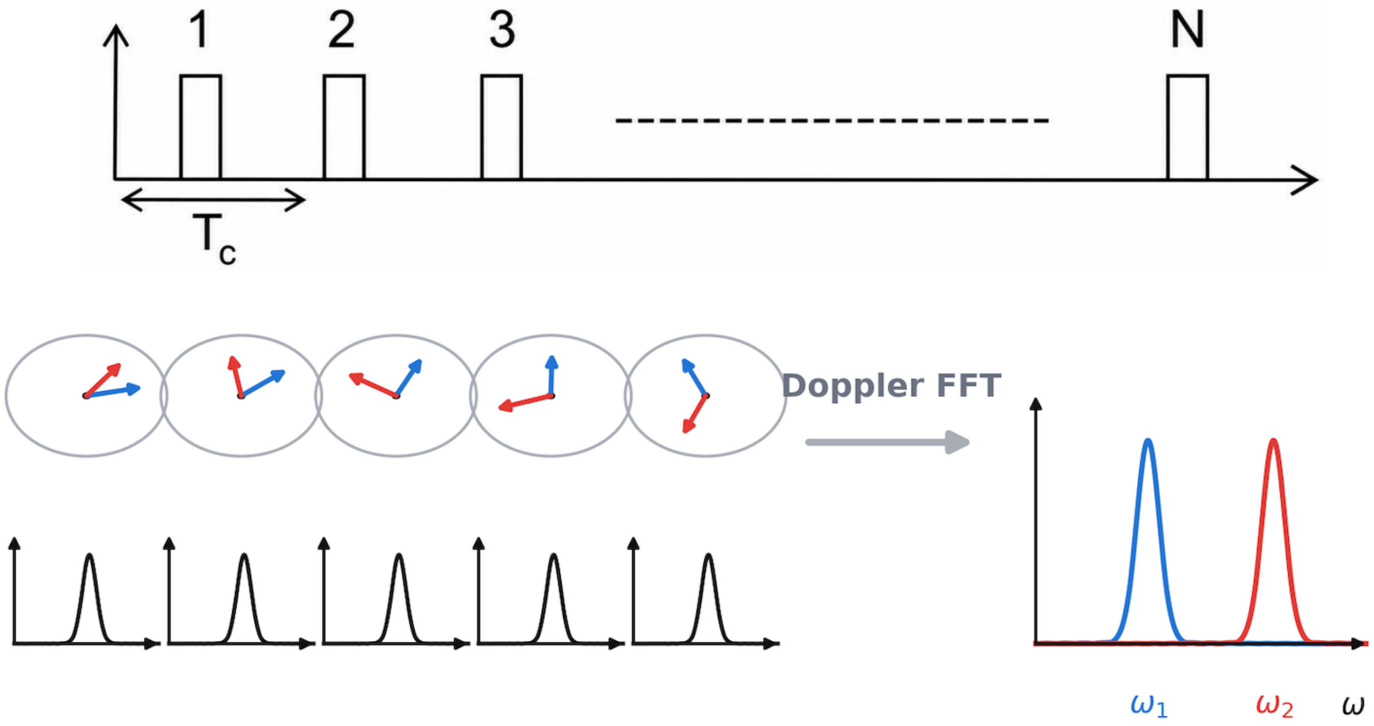
The value at the peak contains phasor components from both objects.

Therefore, the two-pulse phase-difference method cannot separate their velocities.

# Measuring Velocity with Multiple Objects

Transmit N equally spaced pulses.



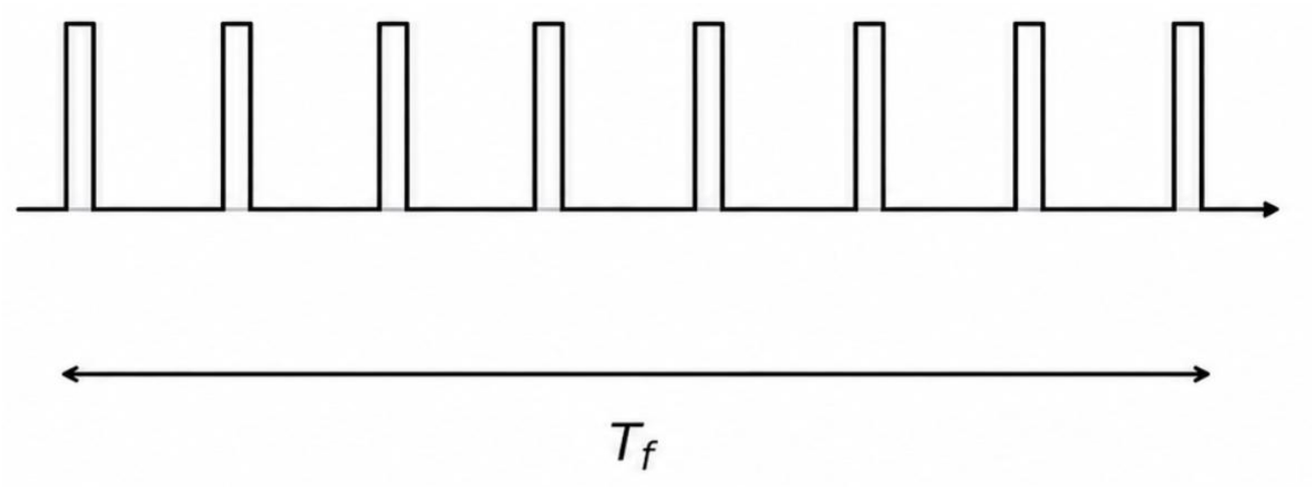

A Doppler FFT across the sequence of phasors resolves the velocities of the two objects.

$$V_1 = \frac{\lambda \omega_1}{4\pi T_c}$$

$$V_2 = \frac{\lambda \omega_2}{4\pi T_c}$$

# Velocity Resolution

A length-N Doppler FFT resolves phase rates only when $|\omega 1 - \omega 2| > 2\pi/N$.




$T_f = NT_c$

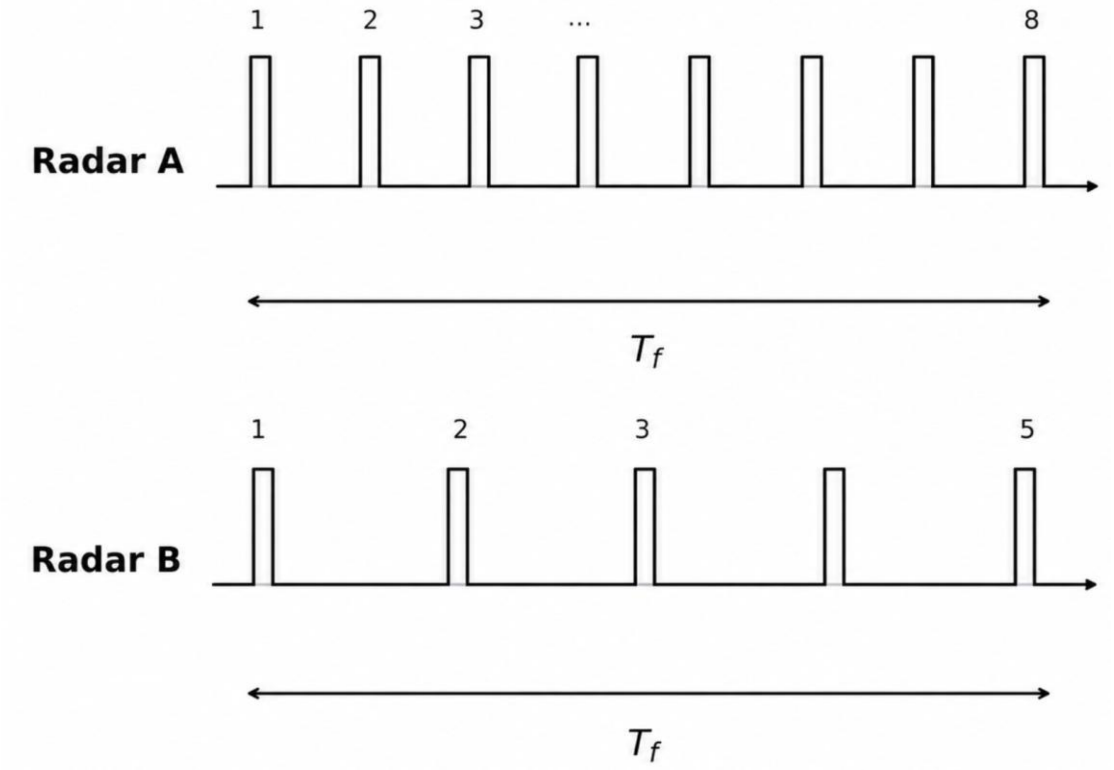
$$\Delta\omega = \frac{4\pi \Delta v T_c}{\lambda}$$

$$\Delta\omega > \frac{2\pi}{N}$$

$$\Delta v > \frac{\lambda}{2N T_c}$$

$$v_{\text{res}} = \frac{\lambda}{2T_f}$$

# Velocity Resolution



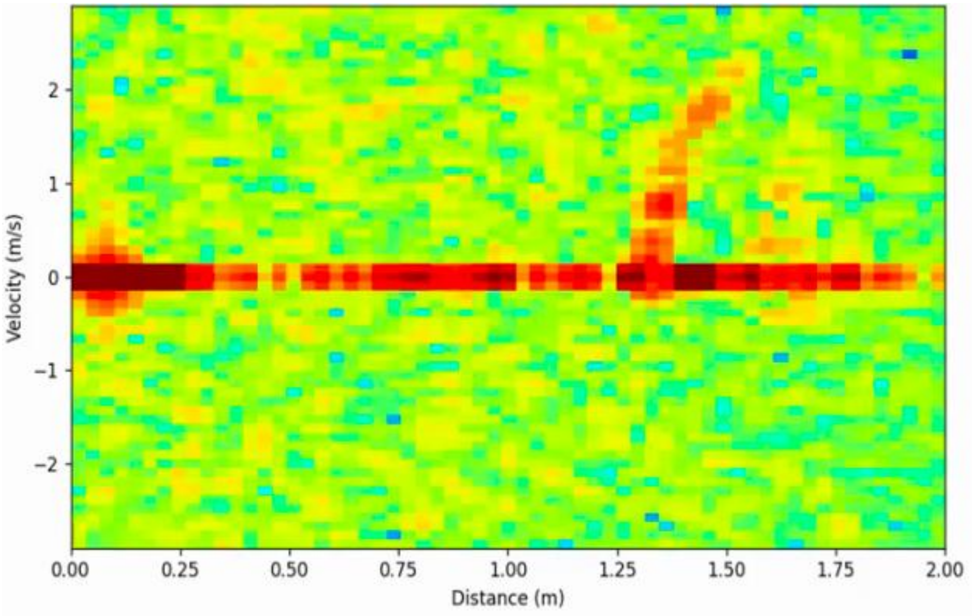

2N equally spaced pulses in one frame.

N equally spaced pulses over the same frame time.

How do Vmax and Vres compare for the two radars?

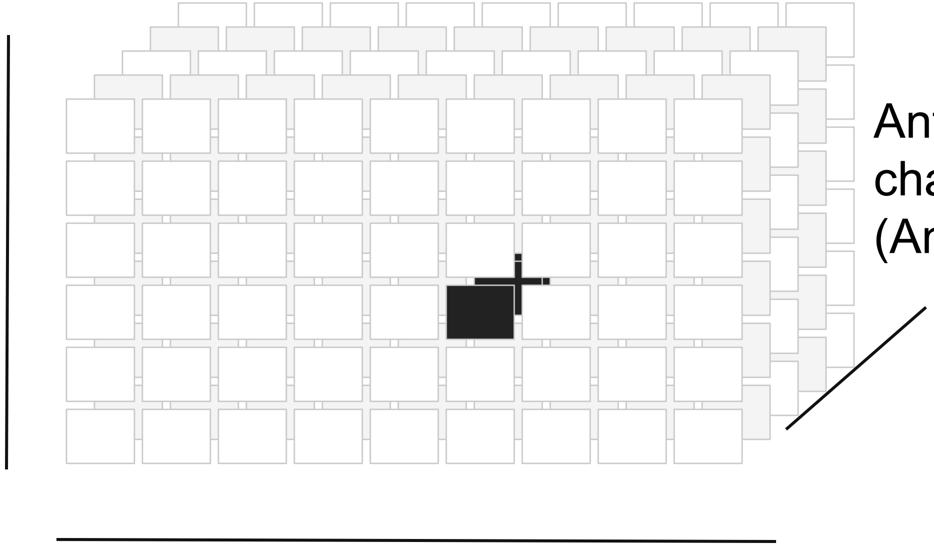
# Range Doppler Map and Radar Data Cube

Doppler




Range

Slow time
Pulses
(velocity)



Antenna
channels
(Angle)

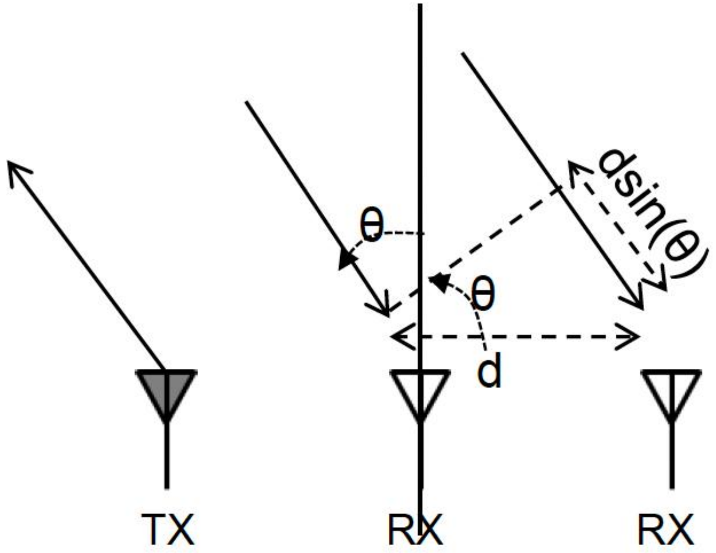
Fast-time samples (Range)

# Angle Estimation

Slides credit: Texas Instruments

# AoA Estimation for One Object

Phase changes across two receivers encode the target's angle.

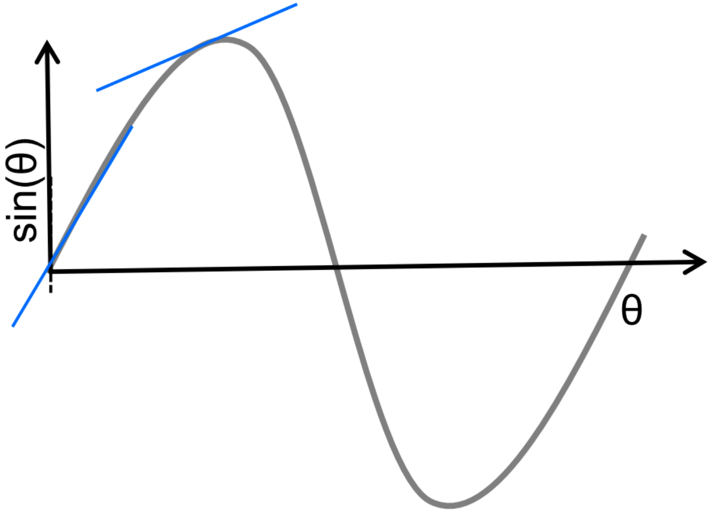



$$\Delta\phi = \frac{2\pi d \sin\theta}{\lambda}$$

$$\theta = \arcsin\left(\frac{\lambda\,\Delta\phi}{2\pi d}\right)$$

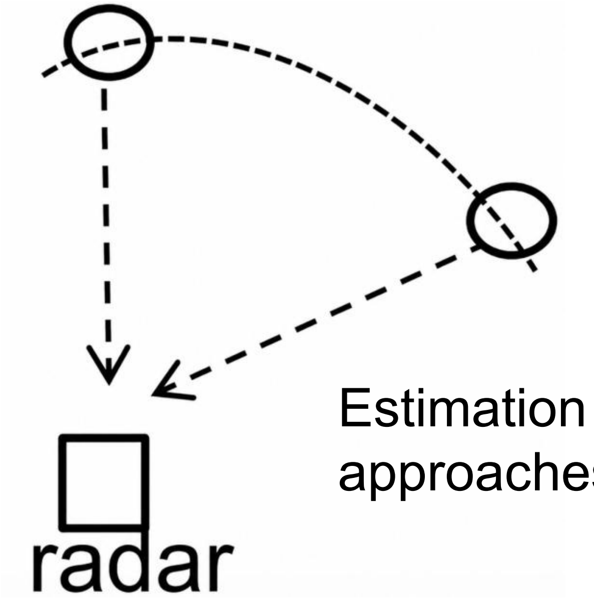
# AoA Estimation Accuracy

Sensitivity of sin(θ) to θ degrades as θ increases



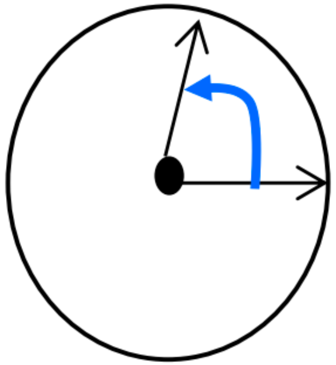

$$\Delta\phi = \frac{2\pi d \sin\theta}{\lambda}$$

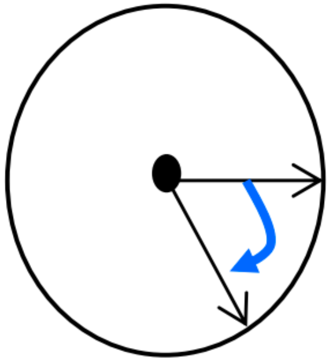
At θ =0, ω is most sensitive to changes in θ.

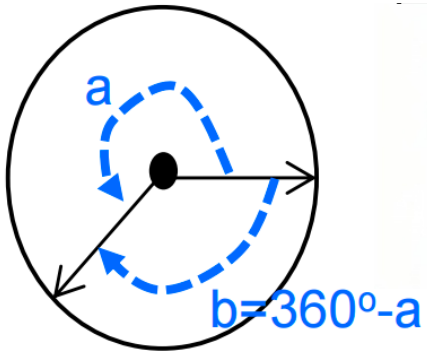



Estimation accuracy degrades as θ approaches 90 degrees

# Angular Field of View (FoV)



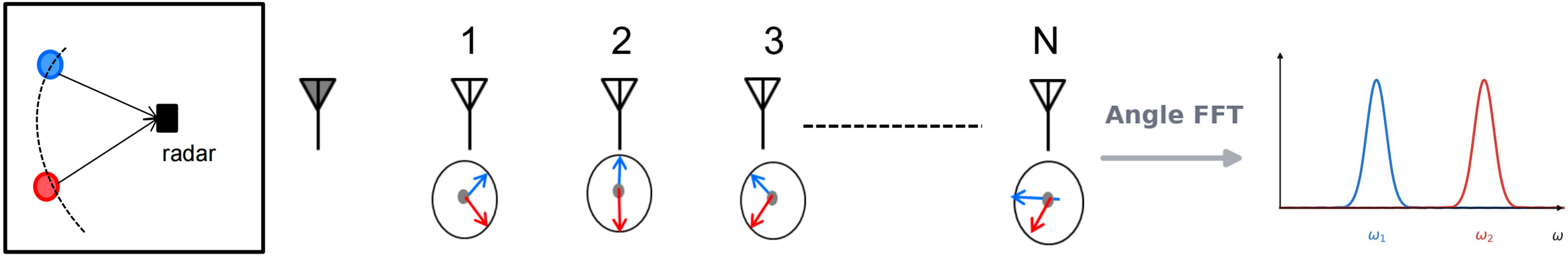

The measurement is unambiguous only if | ω | < π.

$$\left|\frac{2\pi d \sin\theta}{\lambda}\right| < \pi$$

$$|\theta| < \sin^{-1}\left(\frac{\lambda}{2d}\right)$$

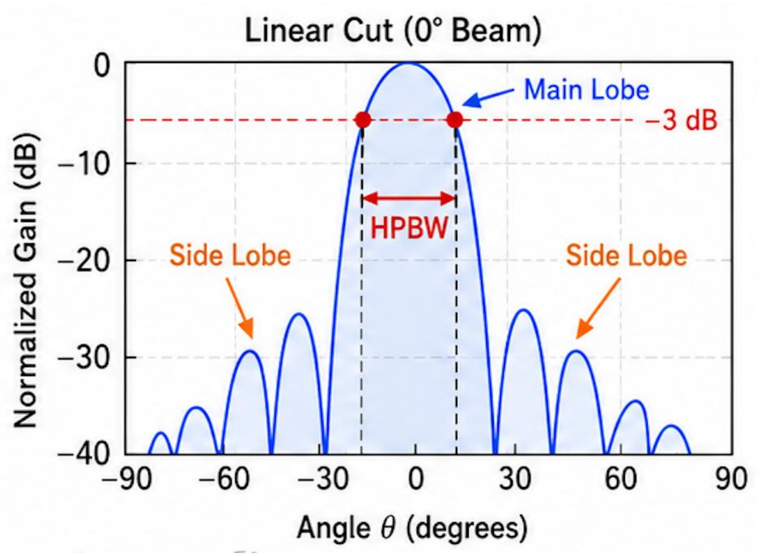
# AoA Estimation for Multiple Objects

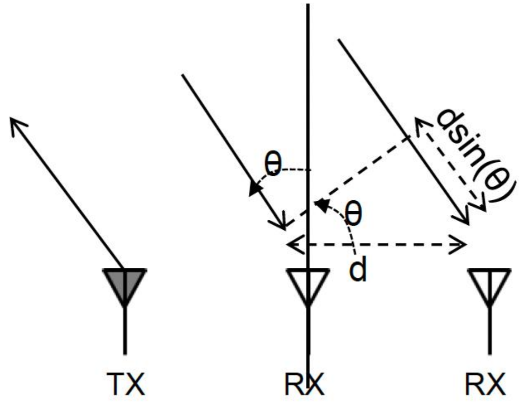
Phase changes across N receivers encode multiple targets’ angles.



# Why Are There Sidelobes?

A finite antenna array is a spatial window.



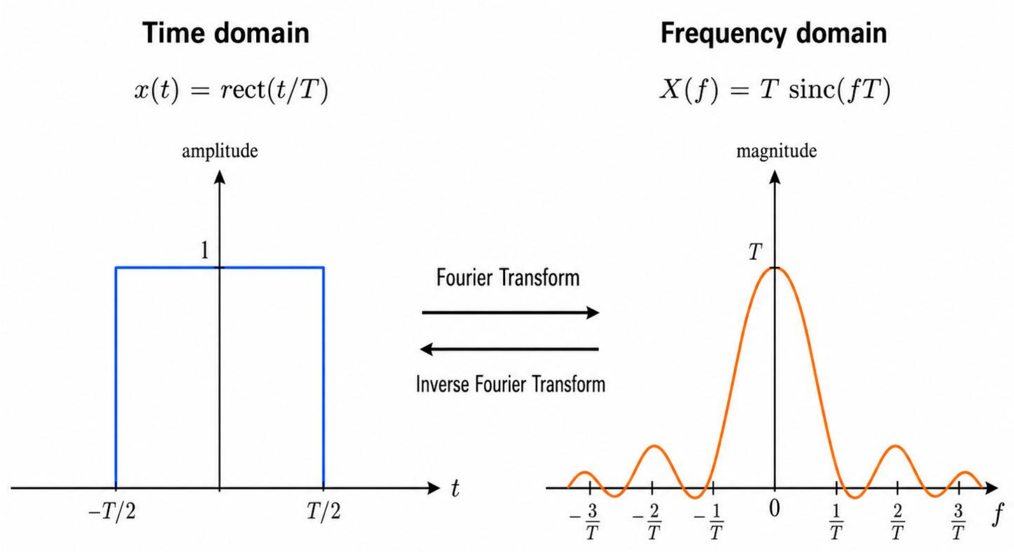

$$AF(\theta) = \sum_{n=0}^{N-1} w_n e^{jnkd\sin\theta}$$

This is equivalent to multiplying the signal by a rectangular window:

$$x_{\text{obs}}(t) = x(t)\,\text{rect}\left(\frac{t}{T}\right)$$

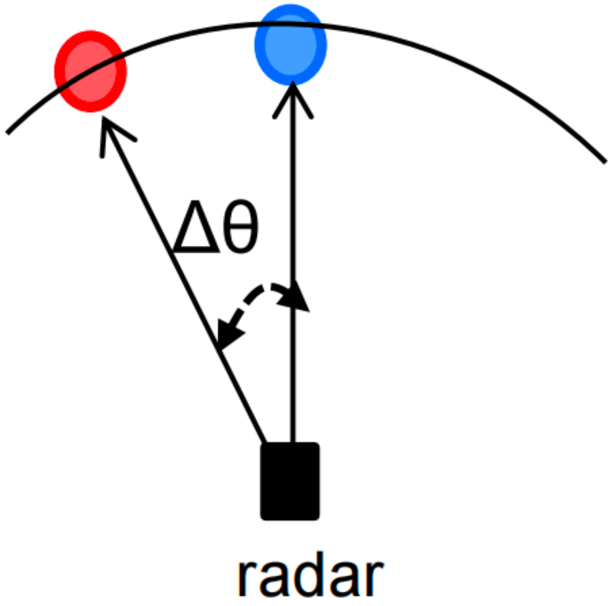
Multiplication in time becomes convolution in frequency:

$$X_{\text{obs}}(f) = X(f) * T\,\text{sinc}(fT)$$



# Angular Resolution

Larger aperture -> Better angular resolution




$$\Delta\omega = \frac{2\pi d}{\lambda}\left[\sin(\theta + \Delta\theta) - \sin(\theta)\right]$$

$$\approx \frac{2\pi d}{\lambda}\cos(\theta)\,\Delta\theta$$

$$\Delta\omega > \frac{2\pi}{N}$$

$$\Rightarrow \frac{2\pi d}{\lambda}\cos(\theta)\,\Delta\theta > \frac{2\pi}{N}$$

$$\Rightarrow \Delta\theta > \frac{\lambda}{N d\cos(\theta)}$$

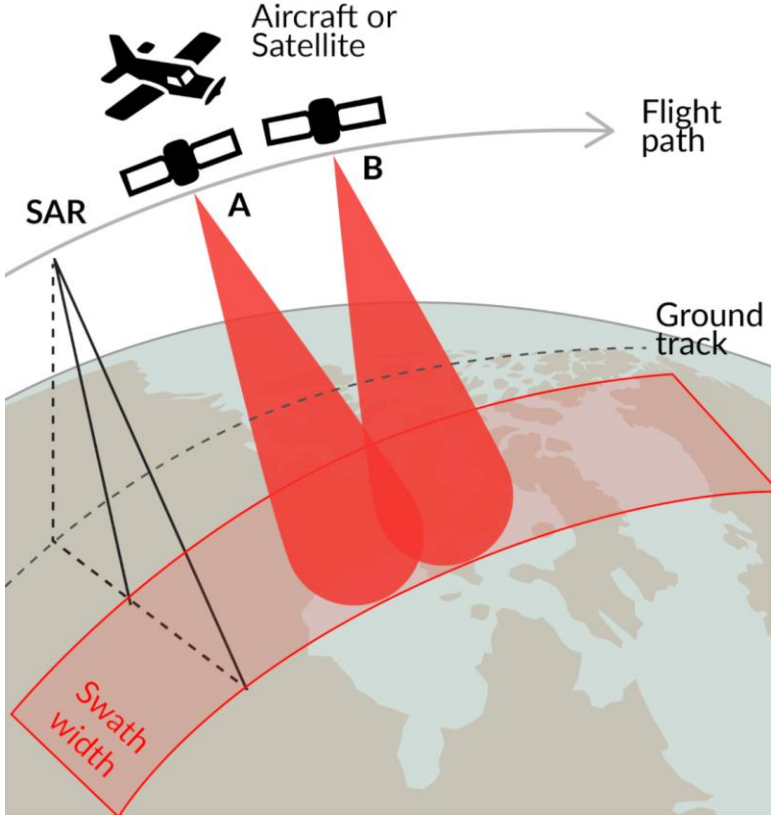
Since derivative of $\sin(\theta)$ is $\cos(\theta)$

$$\frac{\sin(\theta + \Delta\theta) - \sin(\theta)}{\Delta\theta} = \cos(\theta)$$

**How can we improve angular resolution?**

# SAR: Synthetic Aperture Radar

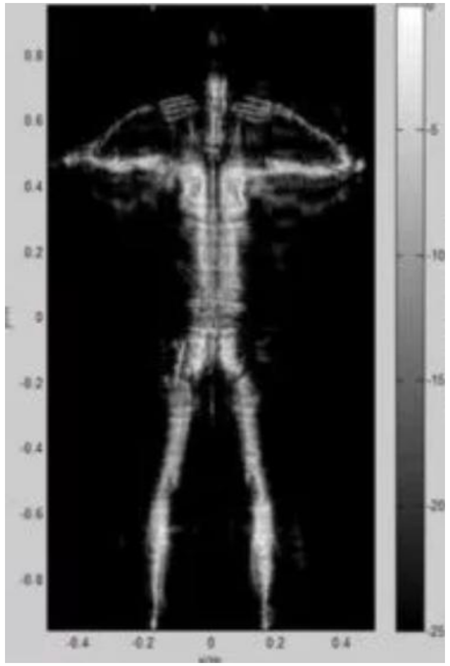
Use antenna motion to create a large virtual aperture.




Spaceborne or airborne SAR for remote sensing

# SAR: Synthetic Aperture Radar

Use antenna motion to create a large virtual aperture.



mmWave near-field SAR
for security check

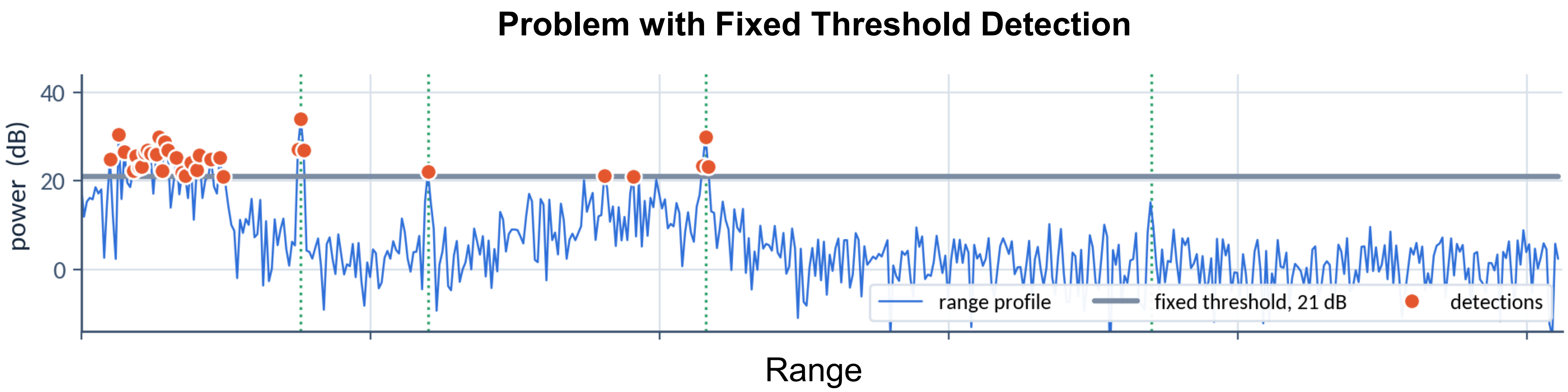
**What are the two fundamental requirements for SAR imaging?**

**Accurate motion tracking and static scene.**

# CFAR Detection

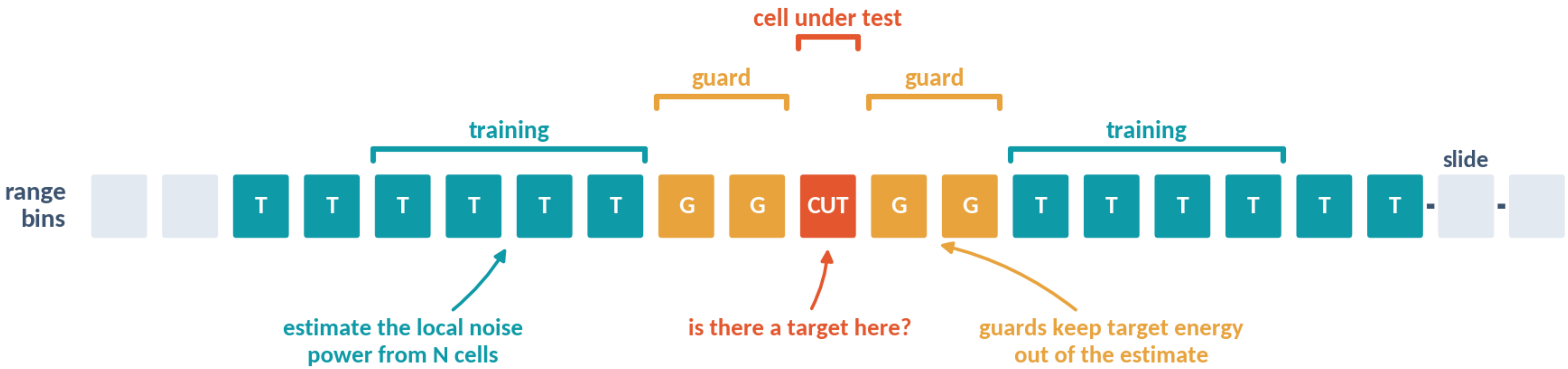
# CFAR Detection

**CFAR: Constant False Alarm Rate**

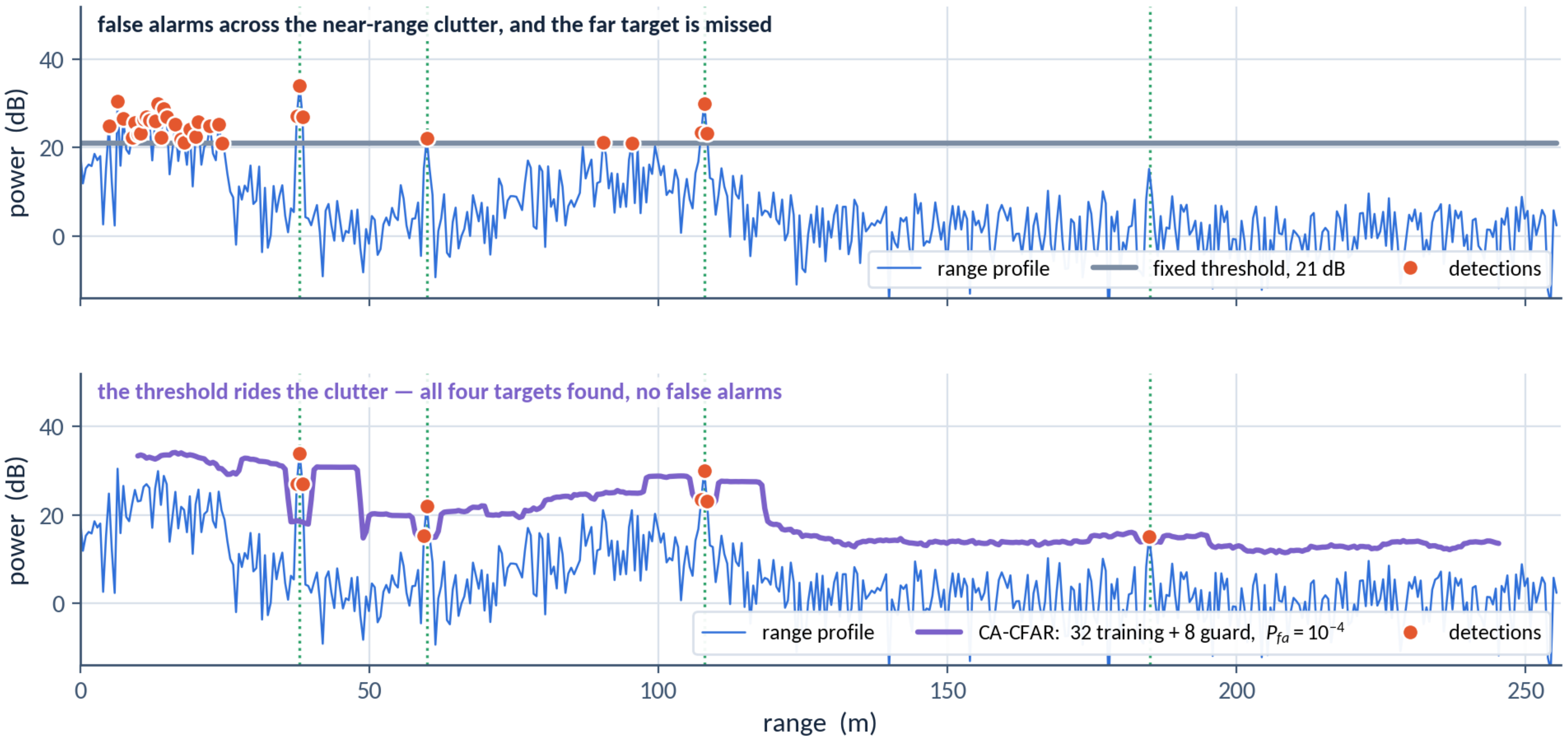



**How does CFAR solve the problem?**

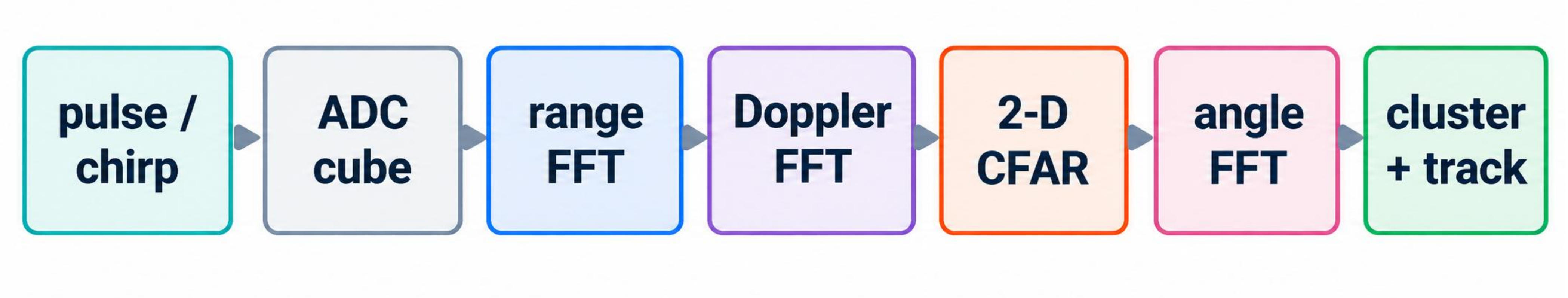
# Cell-Averaging CFAR



# Cell-Averaging CFAR



# Radar Signal Processing Pipeline




**Does the order matter?**

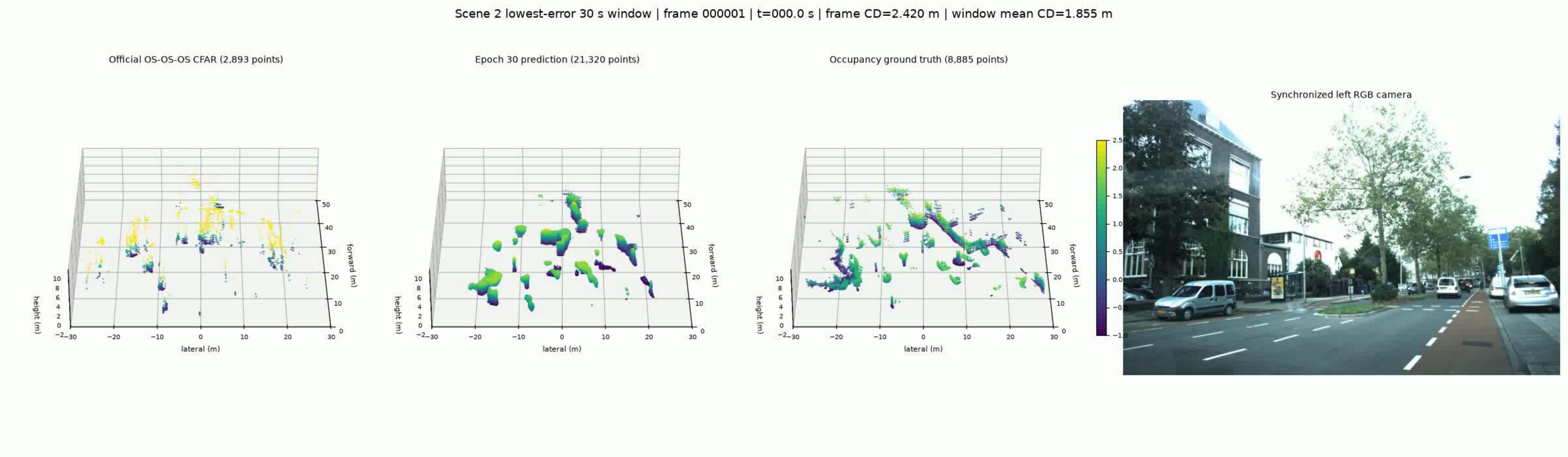
Range FFT is usually done first

Doppler FFT and Angle FFT can sometimes be swapped except TDM-MIMO radar

CFAR is usually before angle estimation for reducing computation

# Point Cloud Generation

OS-CFAR | DNN | LiDAR | Camera



# TwinFocus: Autofocus for Handheld mmWave SAR Imaging via Physical and Digital Twin References

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**University of Washington**

***ACM MobiSys 2026***

**BE BOUNDLESS**

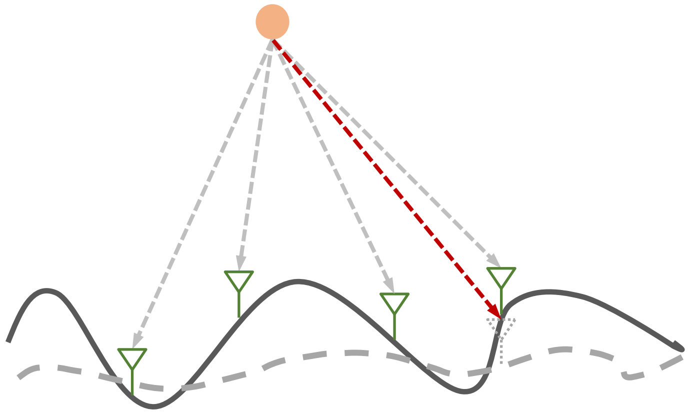
# Can we make high-resolution mmWave imaging mobile and cost-effective?

**Mechanical Scan**
**Bulky and Expensive**


**Handheld Scan**
**Mobile and Cost-effective**

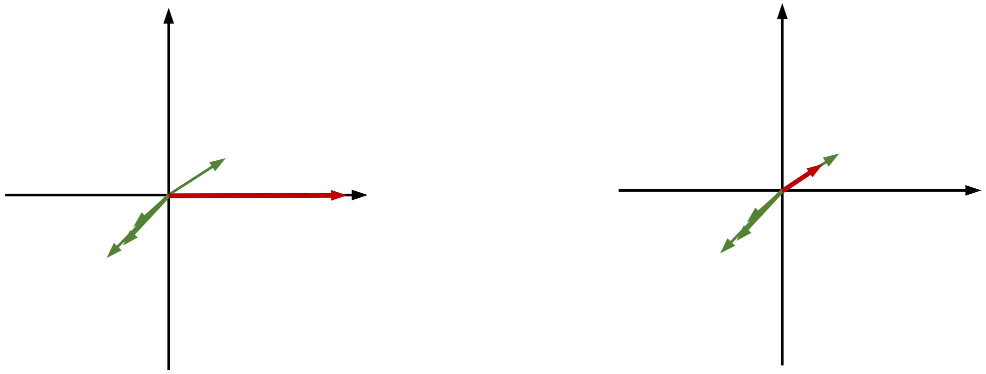
# Challenge: Motion Error Degrades Image Quality

## BackProjection Imaging



BP imaging

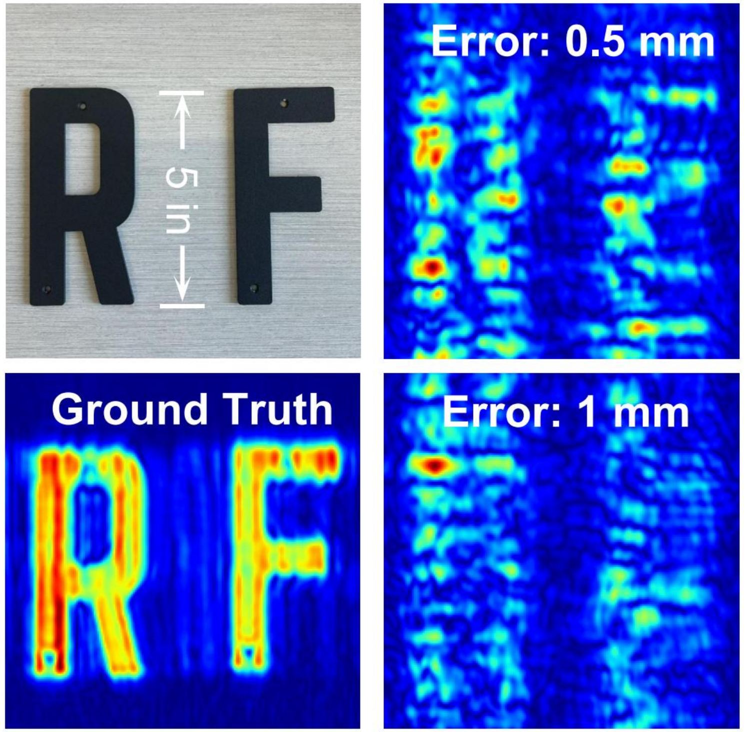
BP with motion error



**Accumulation**

**Cancellation**

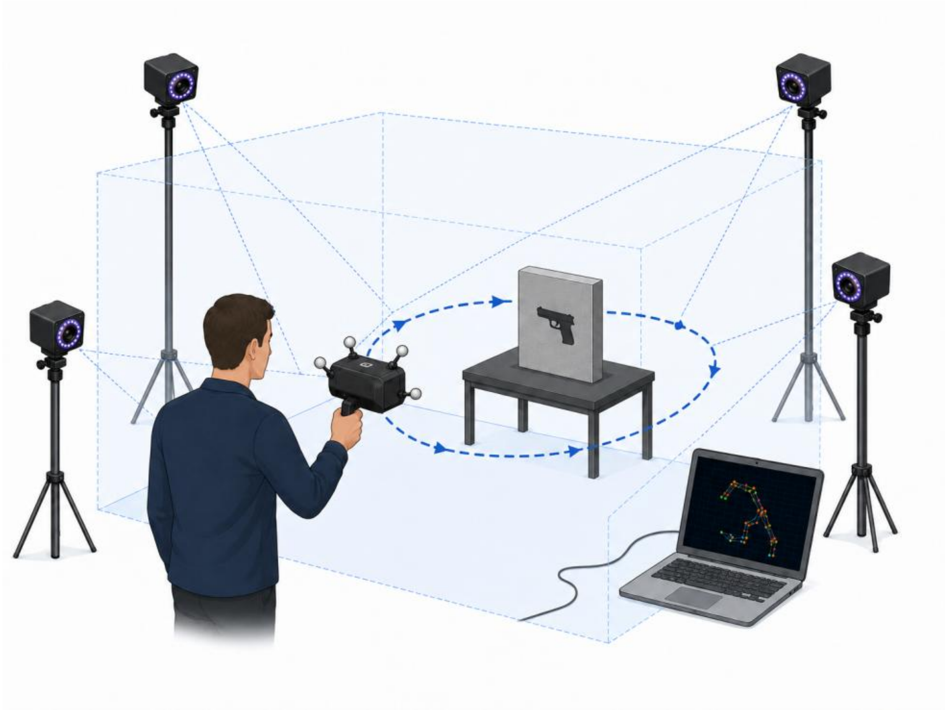
## Impact of Motion Error




For mmWave, even sub-mm motion errors cause image distortion

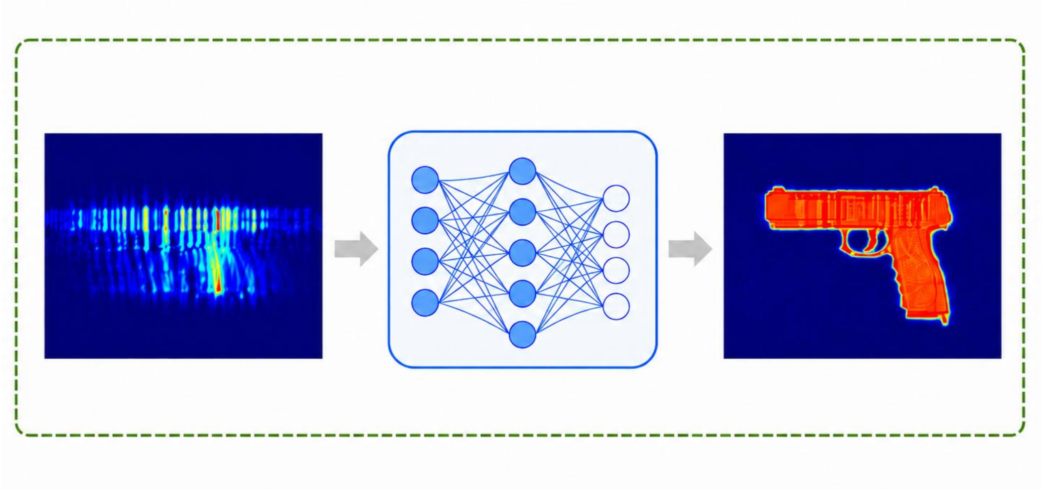
# Limitations of Existing Solutions

## Expensive Motion Compensation



**Requires costly motion capture to achieve precise tracking**

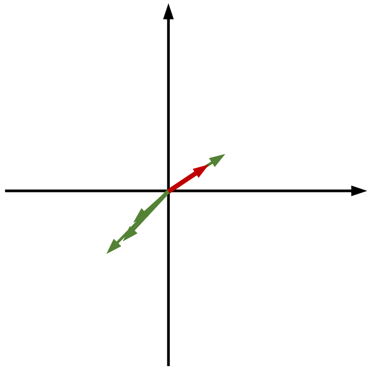
## Data-Hungry Neural Network



**Requires extensive training data to generalize across scenarios**

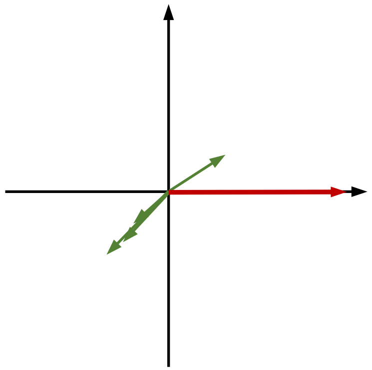
# Autofocus Optimization Design

**Cancellation**

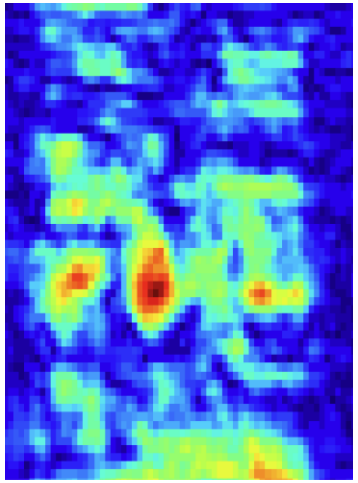


Find optimal phases to compensate for the errors

**Accumulation**



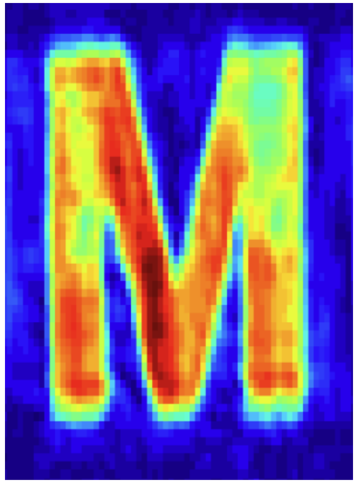
**Prior Level 1: If we know the answer, can we design the optimization to match the answer?**



Phase Compensation

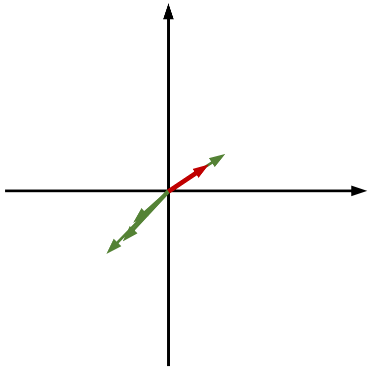
SSIM Cost Function

Gradient Descent



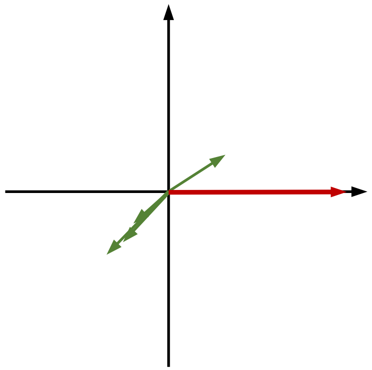
# Autofocus Optimization Design

Cancellation



Find optimal phases to compensate for the errors

Accumulation



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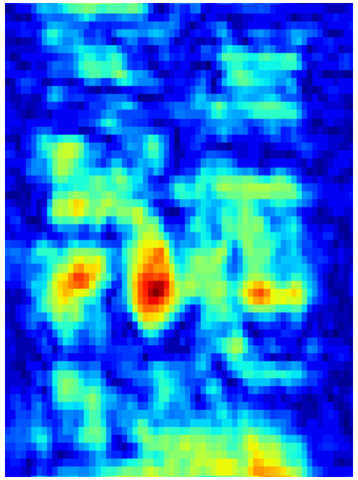
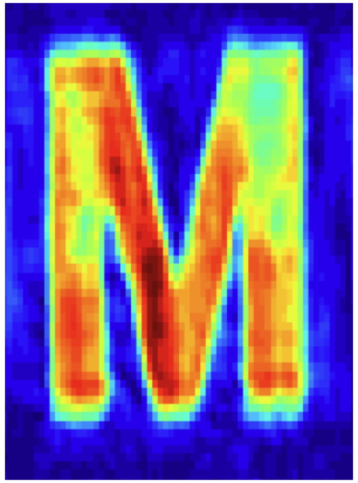


Phase Compensation

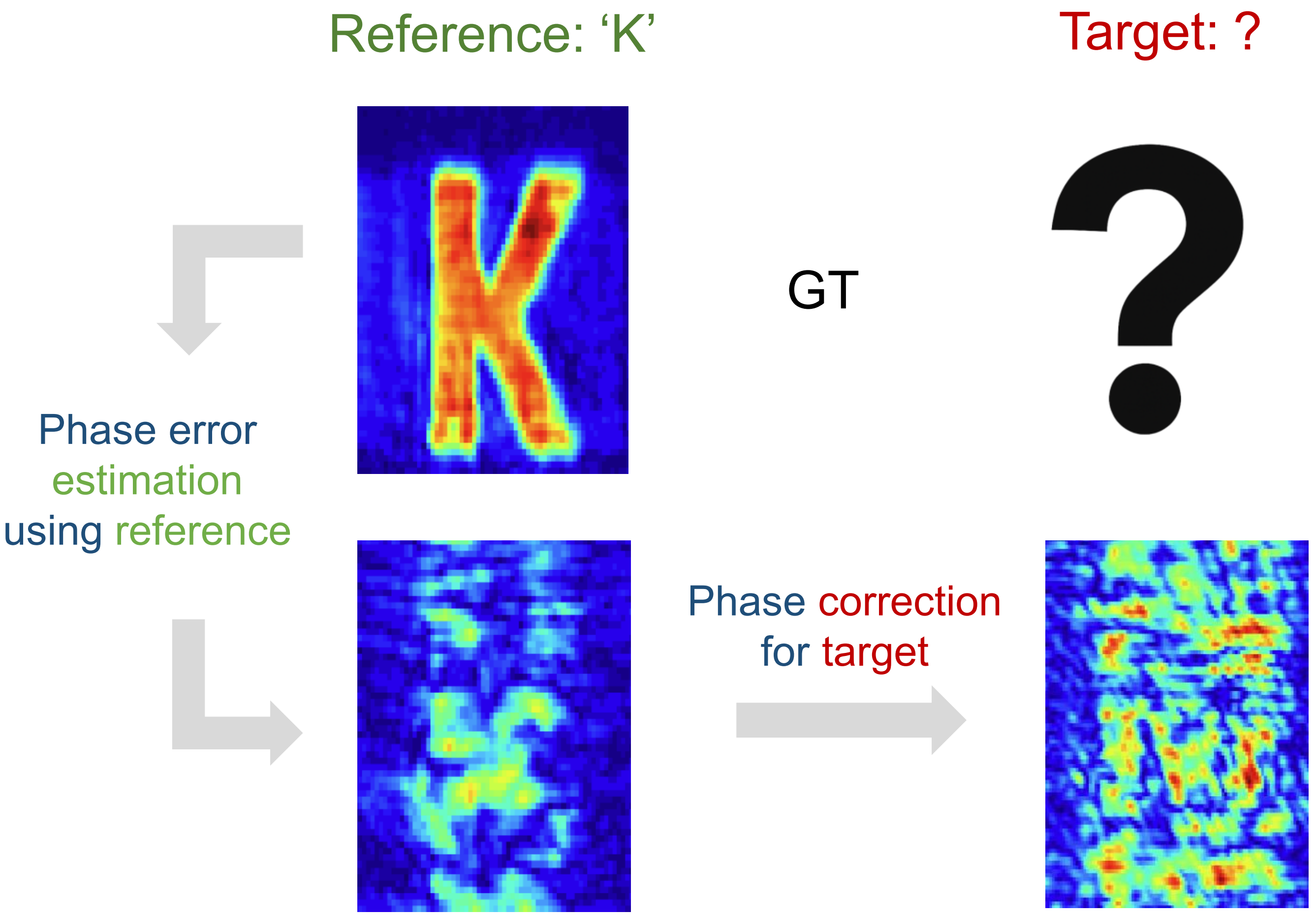
SSIM Cost Function

Gradient Descent



# Autofocus using Physical Twin

## Prior Level 2: If the answer is unknown, can we learn from others’ solutions?



# Autofocus using Physical Twin

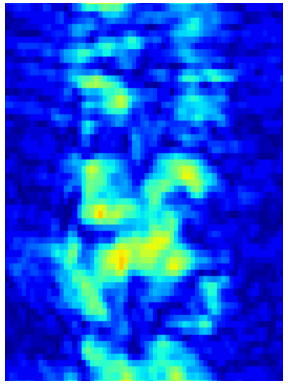
## Prior Level 2: If the answer is unknown, can we learn from others' solutions?

Reference: ‘K’

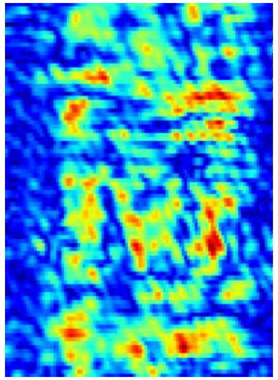
Target: ?

GT

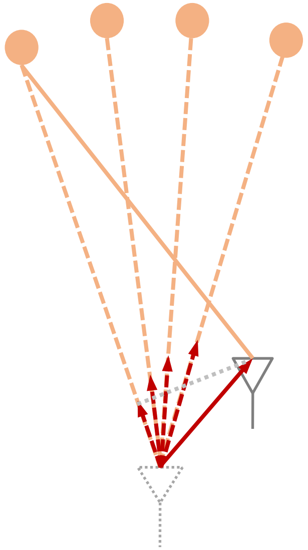
Phase error estimation using reference



Phase correction for target



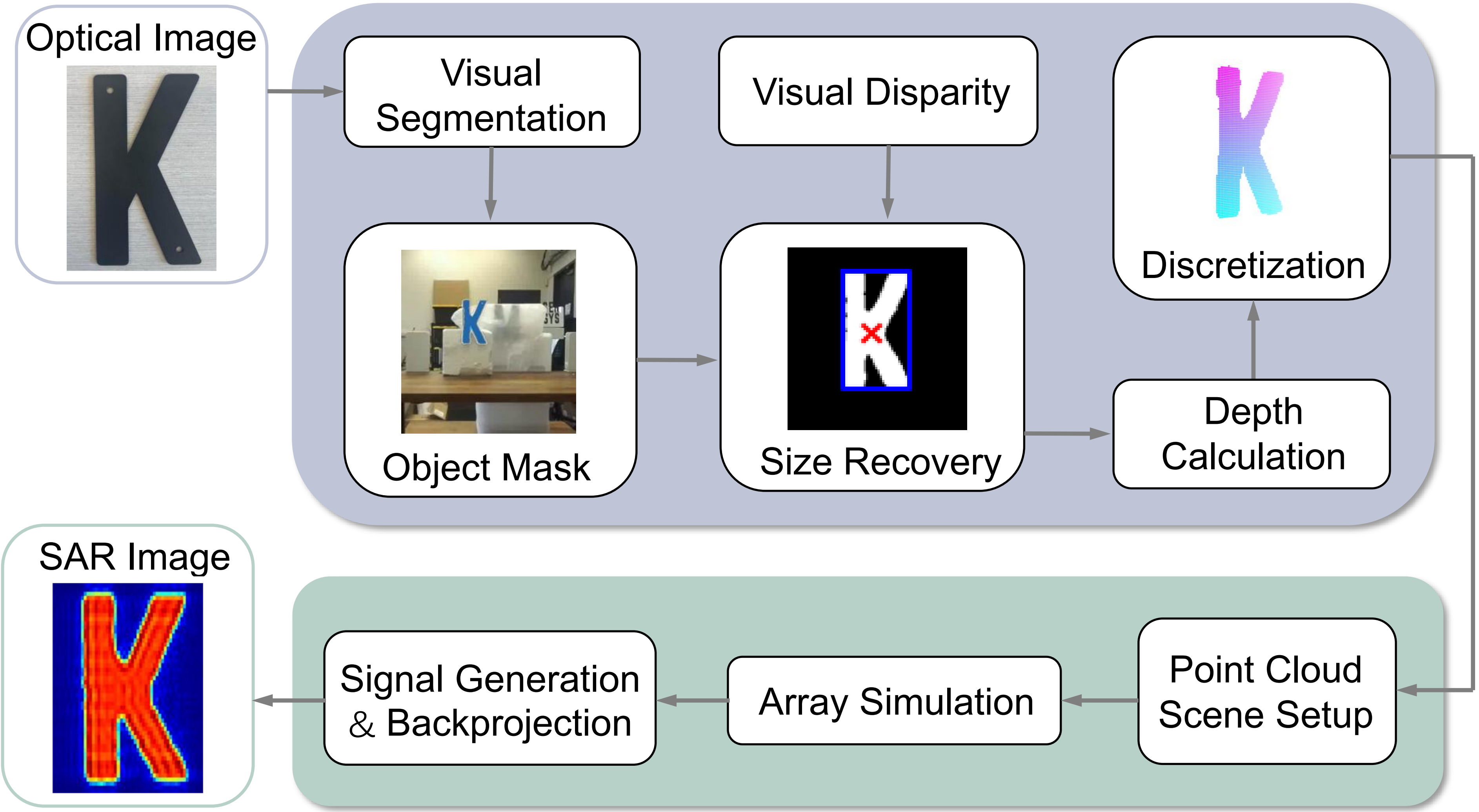
Phase Error Transfer



$$\left|\Delta\phi_1 - \Delta\phi_2\right| \leq \frac{4\pi}{\lambda}\,\theta\,\|\Delta\mathbf{p}\|_2$$

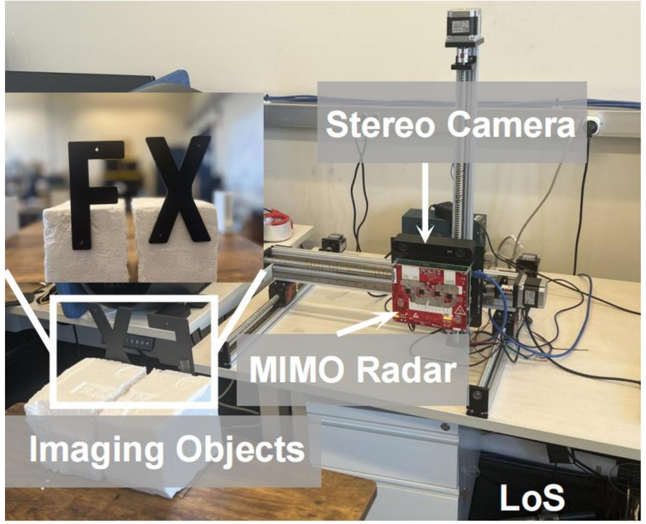
# Autofocus using Digital Twin

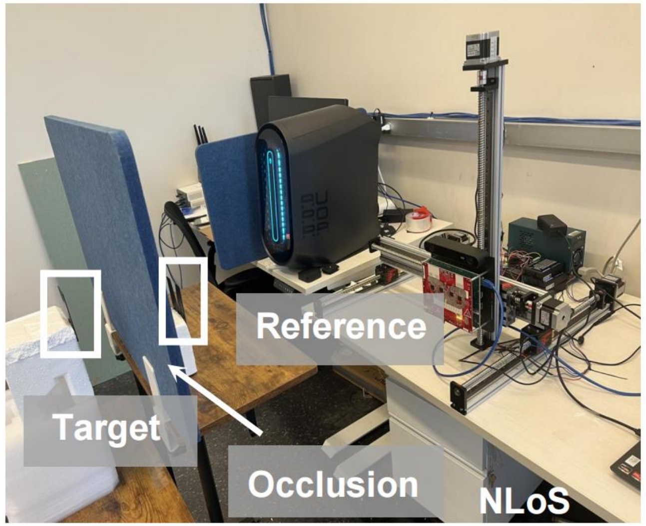
## Prior Level 3: If no answers is available, can we predict the answer using physics?



# Evaluation Setup

- Radar: TI Cascaded Radar (76-81 GHz) captures raw mmWave signals.
- Stereo Camera (ZED 2i): Provides coarse trajectory + 3D reference geometry.
- Motion Stage: moves the radar precisely to collect ground-truth SAR data.



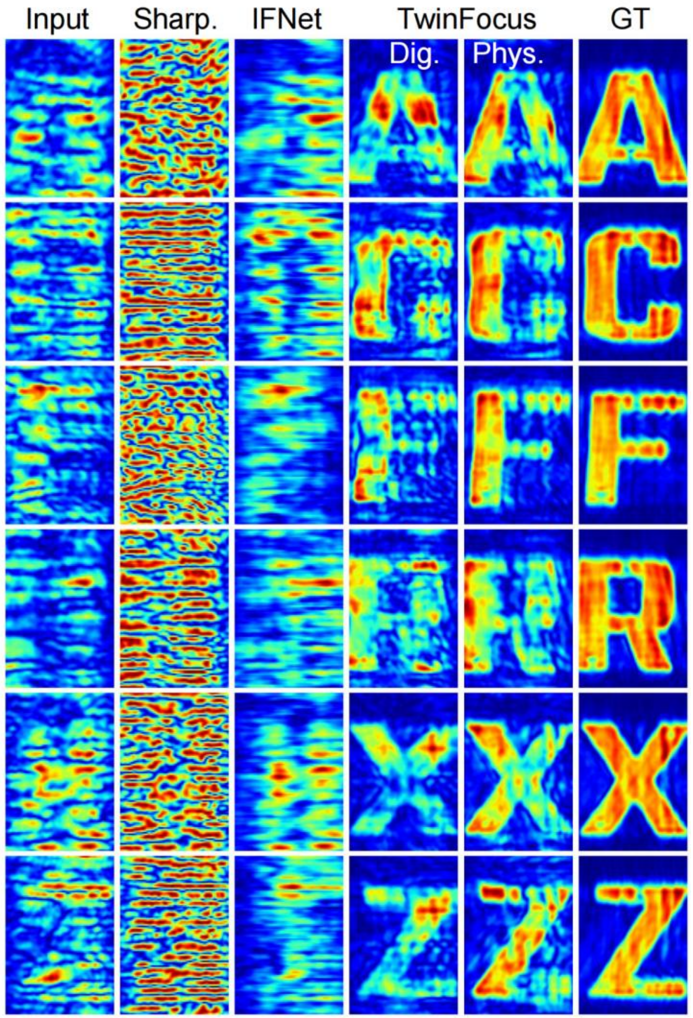




- 18 objects, 38 reference-target pairs (LoS + NLoS).
- Synthetic aperture: 480mm × 260mm.

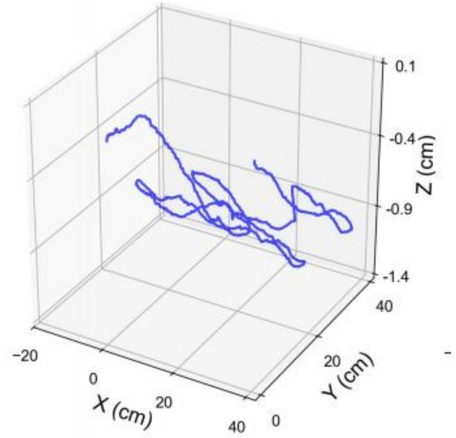
# Evaluation Setup

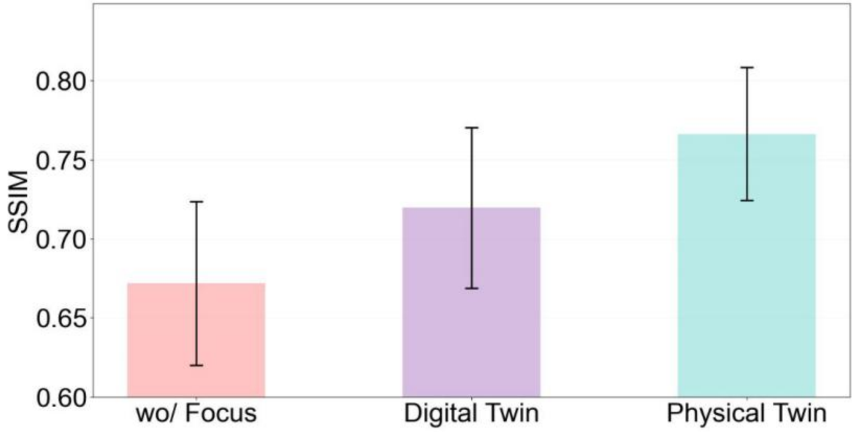
# Experimental Results

## Comparison with Baselines



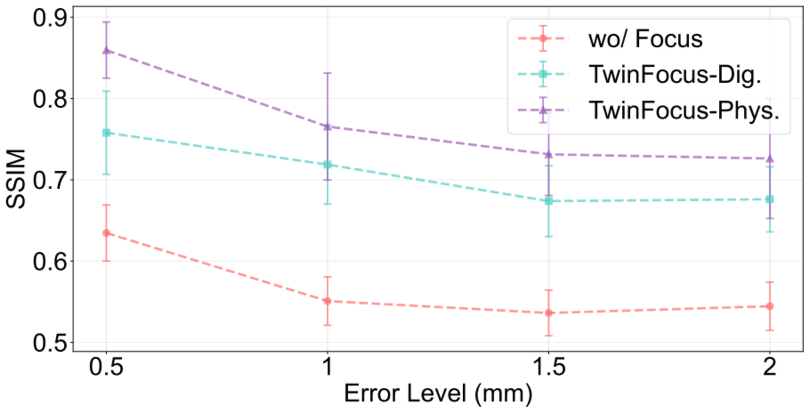

## 3D Handheld Motion



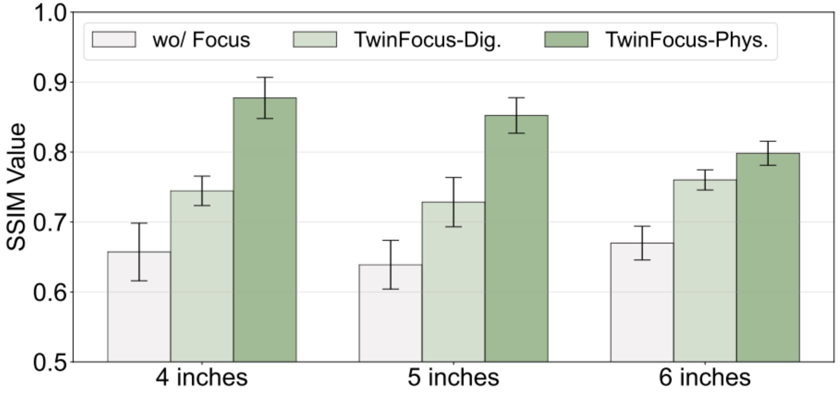



# TwinFocus Robustness

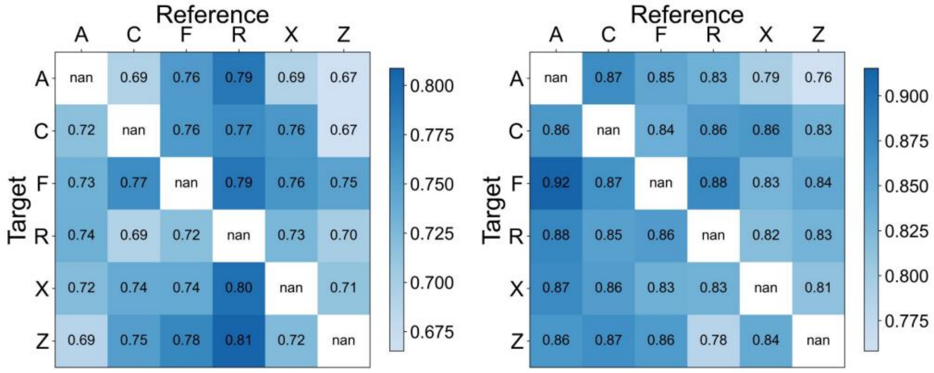
## Tracking Errors




## Object Sizes



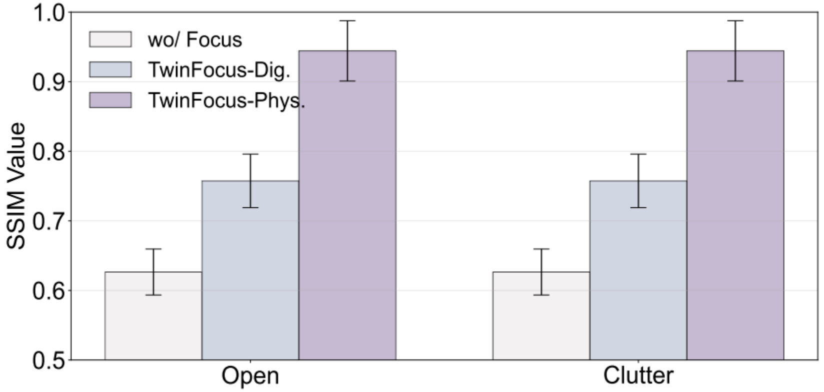

## Object Shapes




(a) SSIM of TwinFocus-Dig.

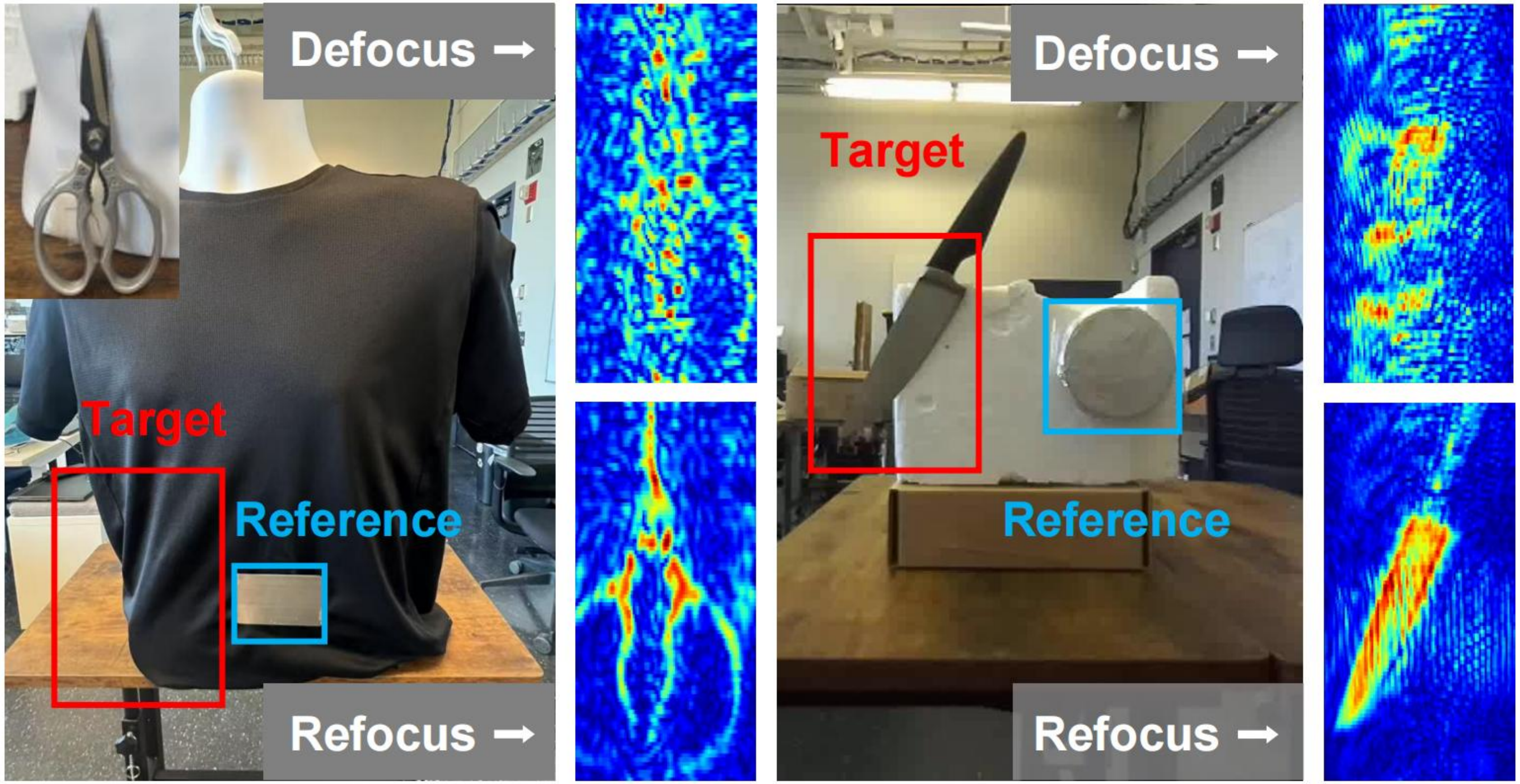
(b) SSIM of TwinFocus-Phys.

## Multipath Environments



# User Study

Security Screening Use-Case Demonstration



# Conclusion & Future Work

- Developed a digital twin-guided autofocus to address sub-mm motion errors in handheld mmWave SAR imaging

- Future work could improve robustness by tolerating larger tracking errors and supporting more complex composite reference objects.

Released Code and Dataset ➡️

TWINFOCUS CODE