18-848 Machine Learning for Radar

Wireless Basics

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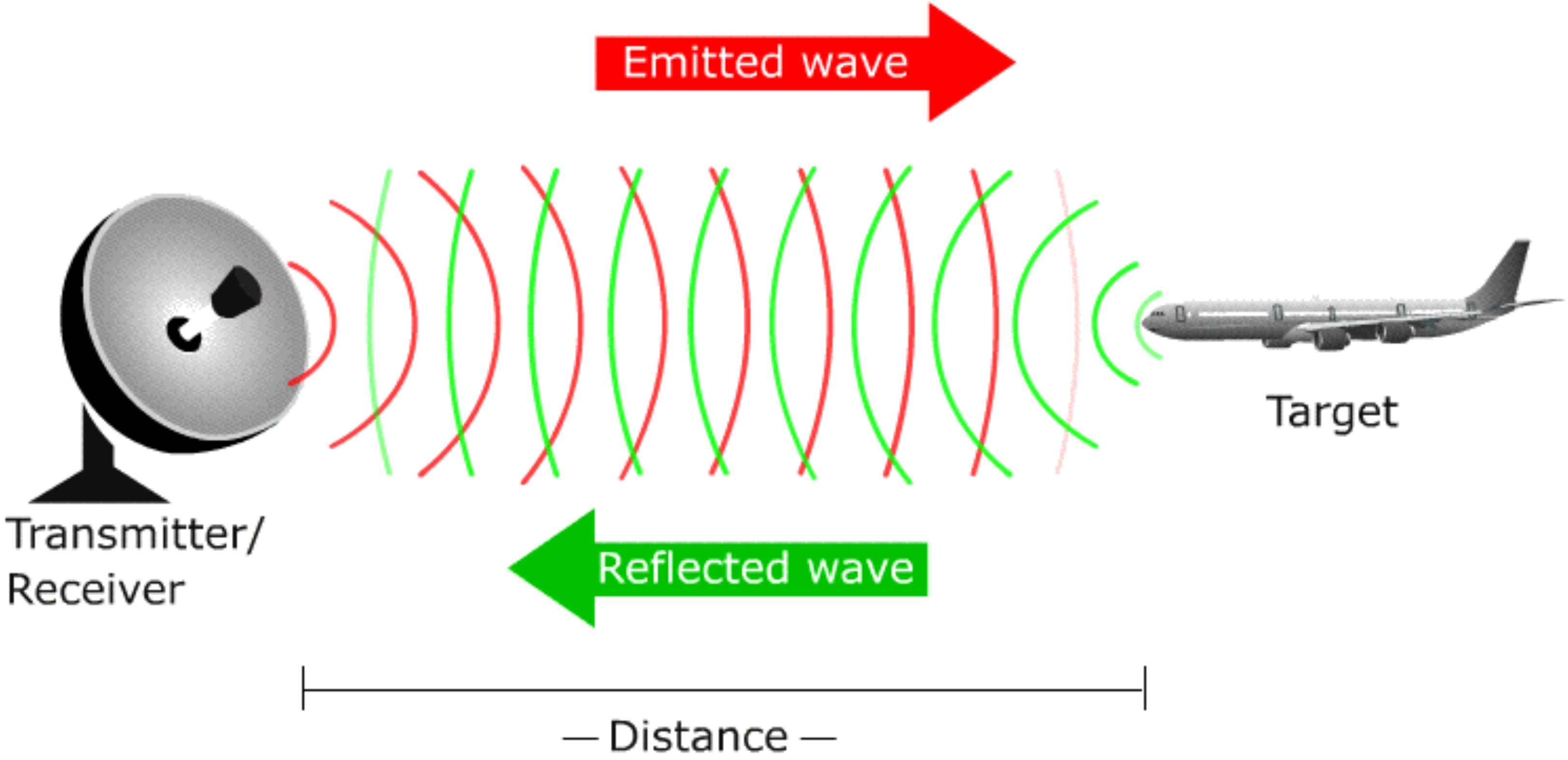
Radar senses with electromagnetic waves

- RAdio Detection And Ranging -> RADAR



Electromagnetism

- Maxwell’s Equation defines how an EM wave is generated and captured.

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$ Electric charge creates electric field.

$$\nabla \cdot \mathbf{B} = 0$$ Single magnetic charge (monopoles) does not exist.

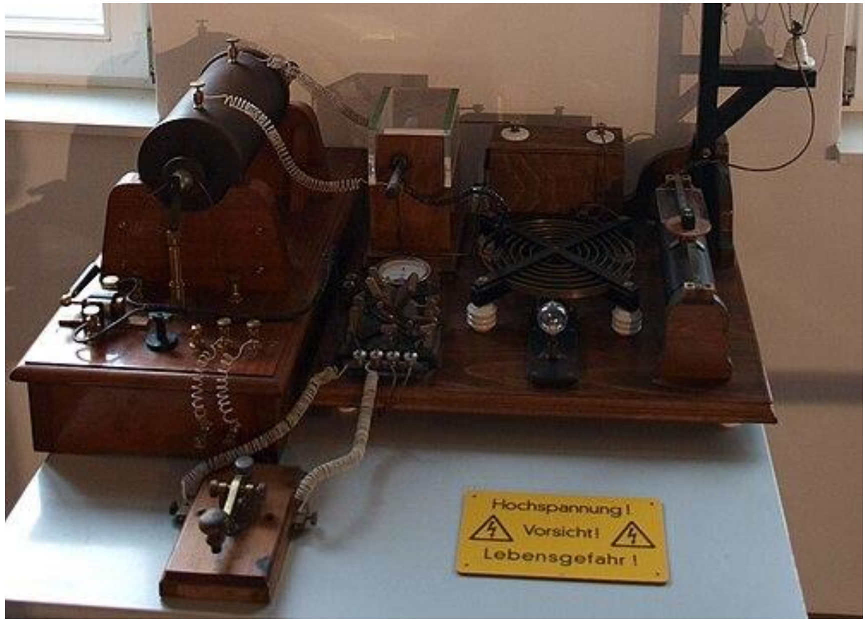
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$ Changing magnetic field creates electric field.

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

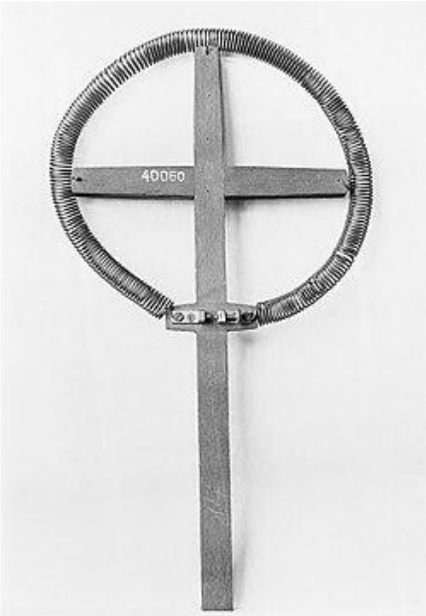
Changing electric field and current creates magnetic field.

Electromagnetic waves

Heinrich Hertz
1857-1894



Oscillator generates an oscillating current.



Antenna generates, sends out, and captures electromagnetic waves.

EM Signal Generator

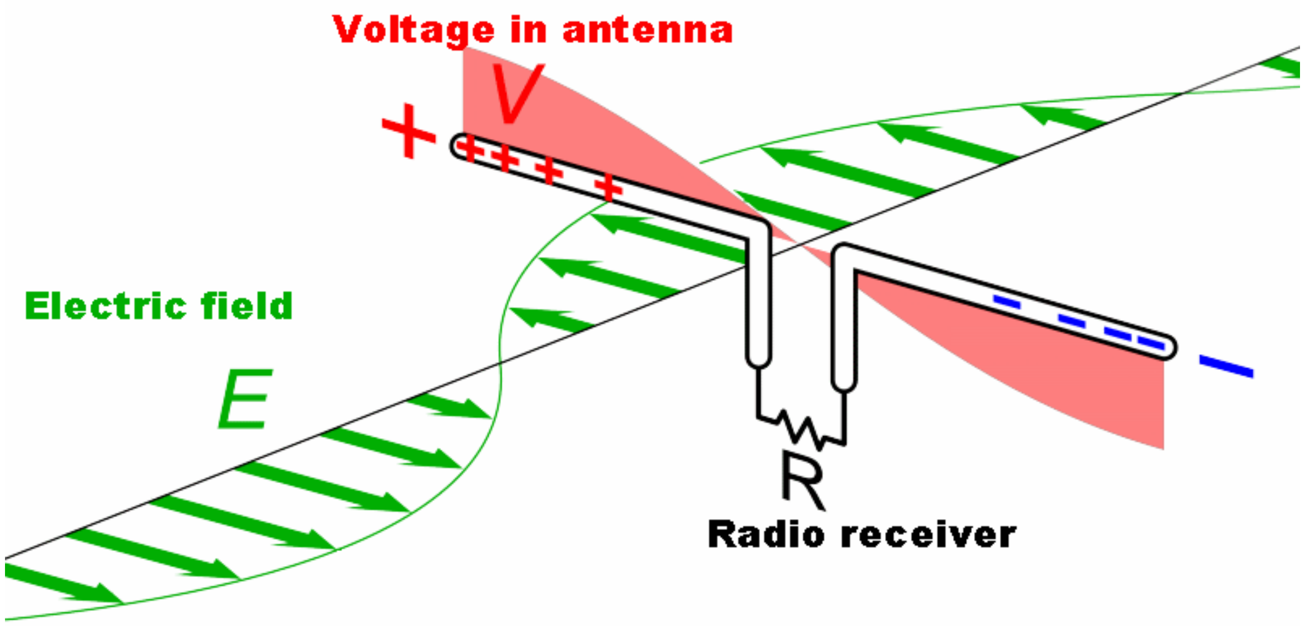
- An **oscillator** is an element that
  - Generates current at a specific frequency, and
  - is usually of a small power.
- An **amplifier** is an element that
  - Makes signals much stronger without hurting them.

Oscillator

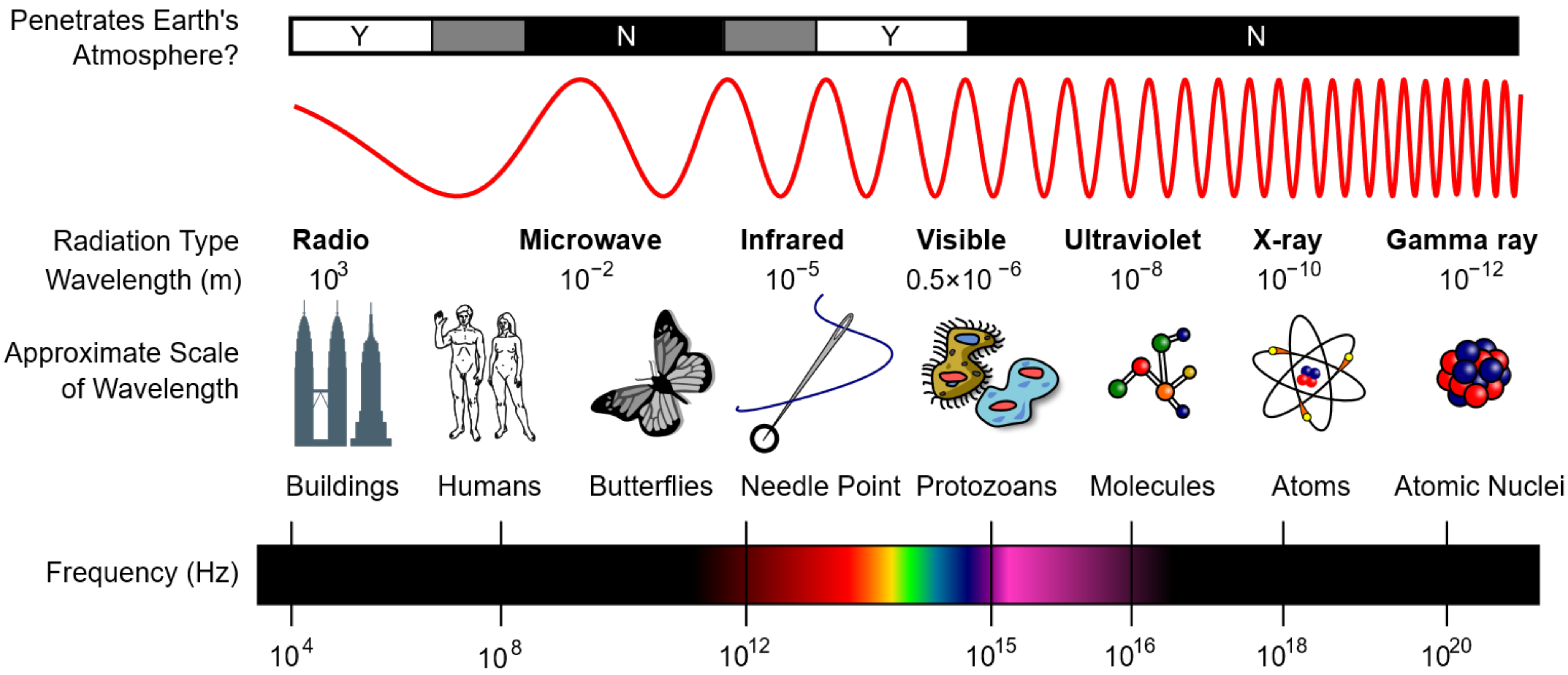
Amplifier

Antenna

- An **antenna** is an element that
  - converts **oscillating current** into **electromagnetic waves**, and
  - converts electromagnetic waves into **oscillating current**.

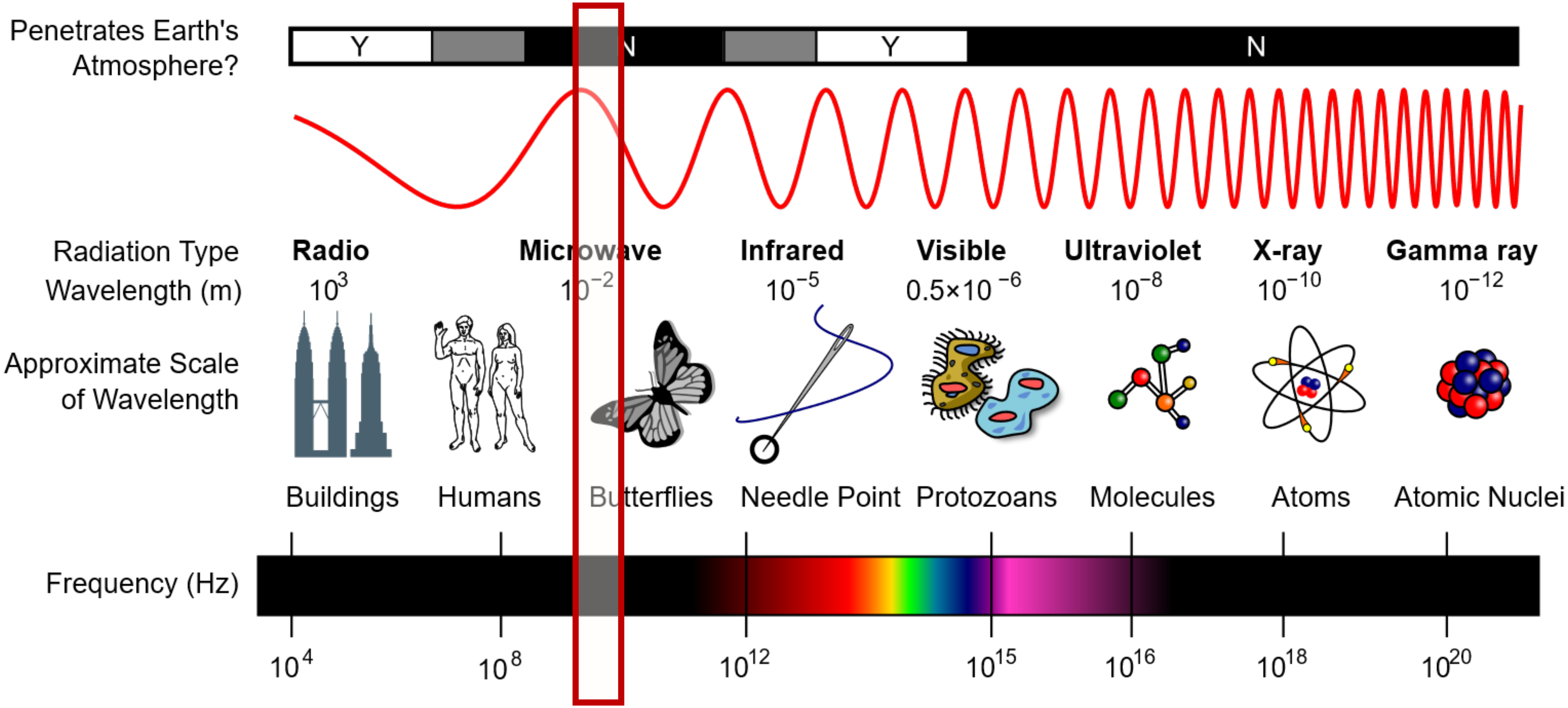


The EM wave spectrum (Wikipedia)



Different frequencies lead to different properties.

The EM wave spectrum (Wikipedia)

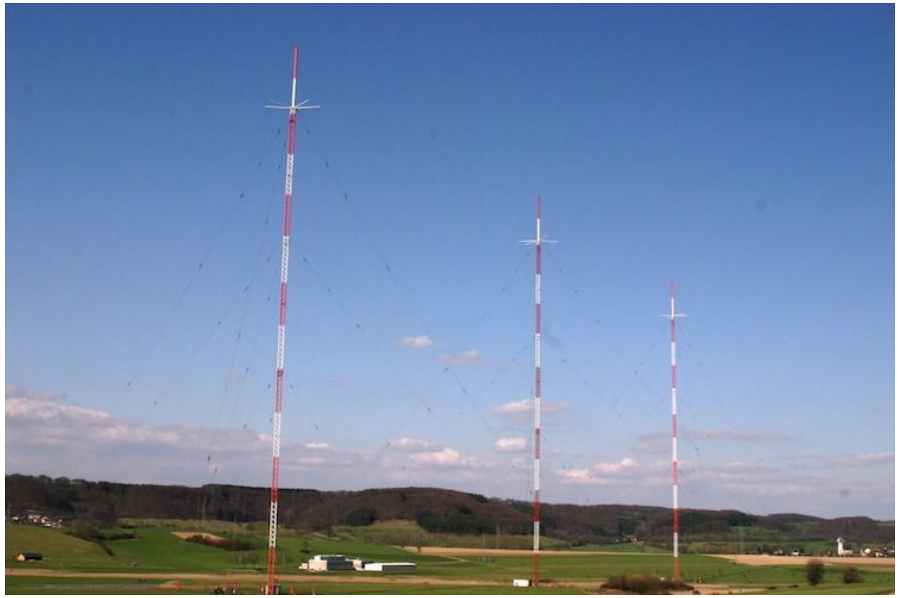


The frequency that we mostly talk about in this course: 1 GHz – 100 GHz.

A comparison table (in the radio spectrum)

Property	Lower frequencies	Higher frequencies
Propagation distance	Larger	Smaller

Propagation Distance



Long wave radios
Frequency=hundreds-thousands kHz
Wavelength=several km
Propagates across the earth.

5G
Frequency=several GHz
Wavelength=several cm
Propagates within a few km.

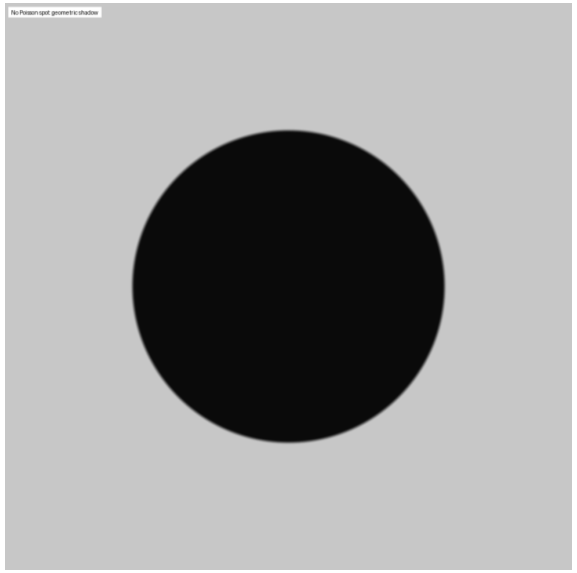
A comparison table (in the radio spectrum)

Property	Lower frequencies	Higher frequencies
Propagation distance	Larger	Smaller
Ability to penetrate through occlusions	Strong	Weak

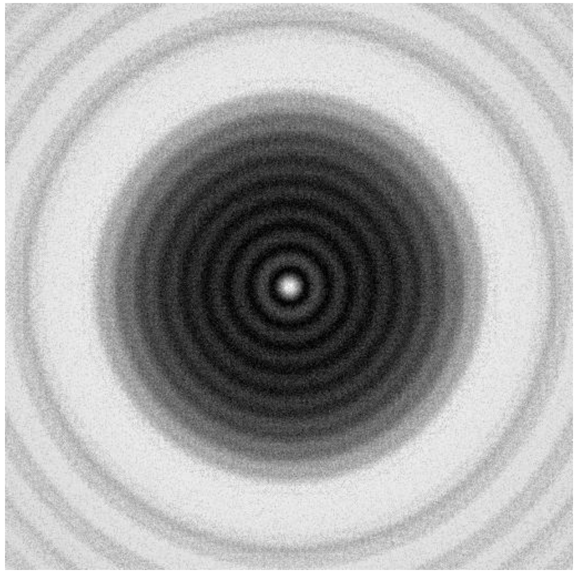
A comparison table (in the radio spectrum)

Property	Lower frequencies	Higher frequencies
Propagation distance	Larger	Smaller
Ability to penetrate through occlusions	Strong	Weak
Behaves like	Wave	Beam

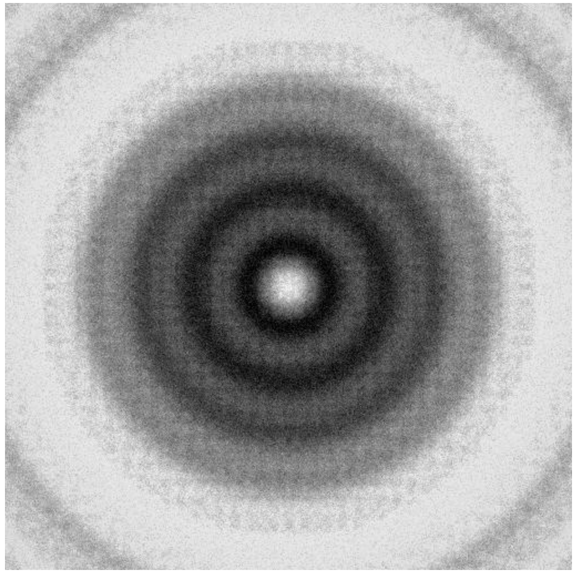
Effect of diffraction at different frequencies



Very high frequency
No diffraction



High frequency
Weak diffraction



Low frequency
Strong diffraction

A comparison table (in the radio spectrum)

Property	Lower frequencies	Higher frequencies
Propagation distance	Larger	Smaller
Ability to penetrate through occlusions	Strong	Weak
Behaves like	“Wave”	“Particle” / “Beam”
Directivity	Usually omni-directional	Usually directional

A comparison table (in the radio spectrum)

Property	Lower frequencies	Higher frequencies
Propagation distance	Larger	Smaller
Ability to penetrate through occlusions	Strong	Weak
Behaves like	“Wave”	“Particle” / “Beam”
Directivity	Usually omni-directional	Usually directional
Bandwidth	Small (Carries less information)	Large (Carries more information)

We will come back to what “bandwidth” means.

Microwave and millimeter wave

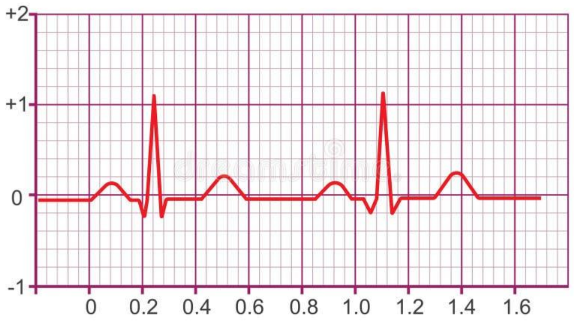
Property	Estimated values
Propagation distance	Tens of meters to kilometers.
Ability to penetrate through occlusions	Can penetrate clothes, wood, dry walls, etc, depending on the thickness.
Behaves like	Mostly like beams, but in some cases like waves.
Directivity	From a few degrees to tens of degrees.
Bandwidth	Tens of MHz to several GHz.

Fourier analysis and frequency domain

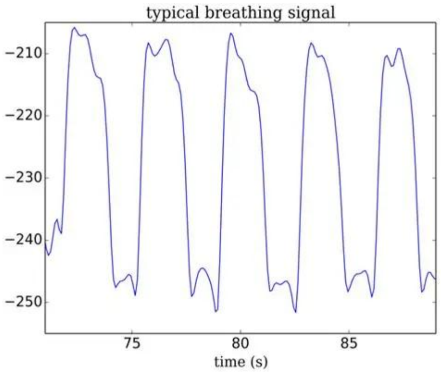
A lot of events that we encounter in life is **periodic.**

$$\exists T, \forall t, x(t) = x(t+T)$$

T is called a **period**.



Heartbeat



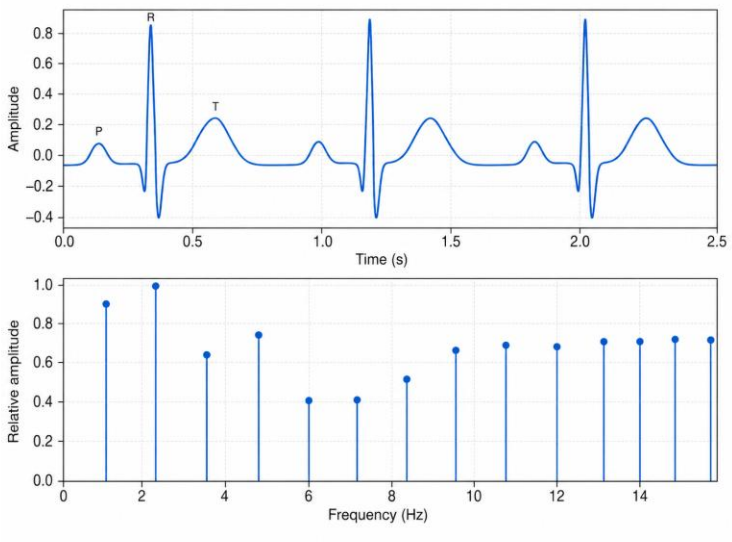
Breathing

Spinning wheel

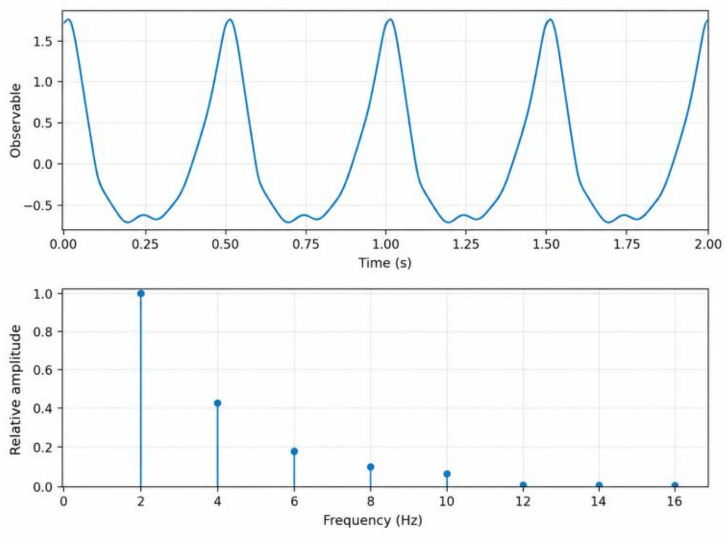
Fourier Series (FS)

Any “well-behaved” **periodic** functions can be written in a **Fourier Series** form.

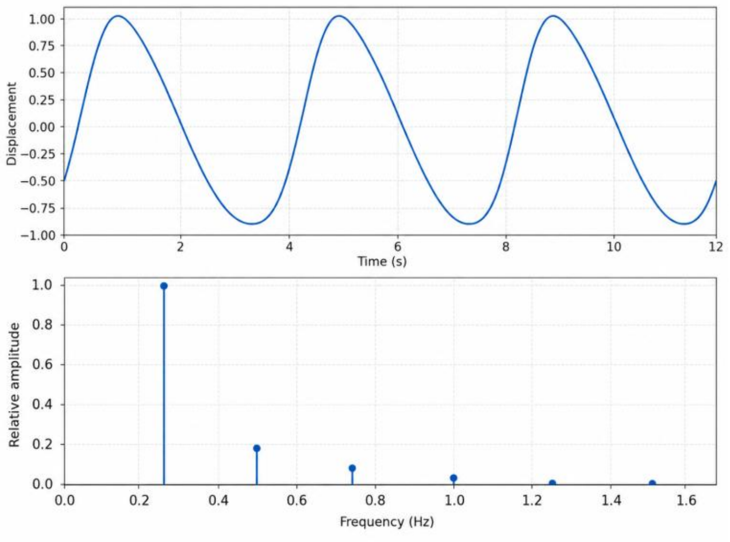
$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j2\pi k f_0 t}$$



Heartbeat



Breathing



Wheel spinning

Continuous-Time Fourier Transform (CTFT)

Not all signals are periodic.

Even for periodic signals, when noise is added, they become non-periodic.

To get frequency components for those signals, we use **Fourier Transform**:

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} \mathrm{d}t$$

We also write

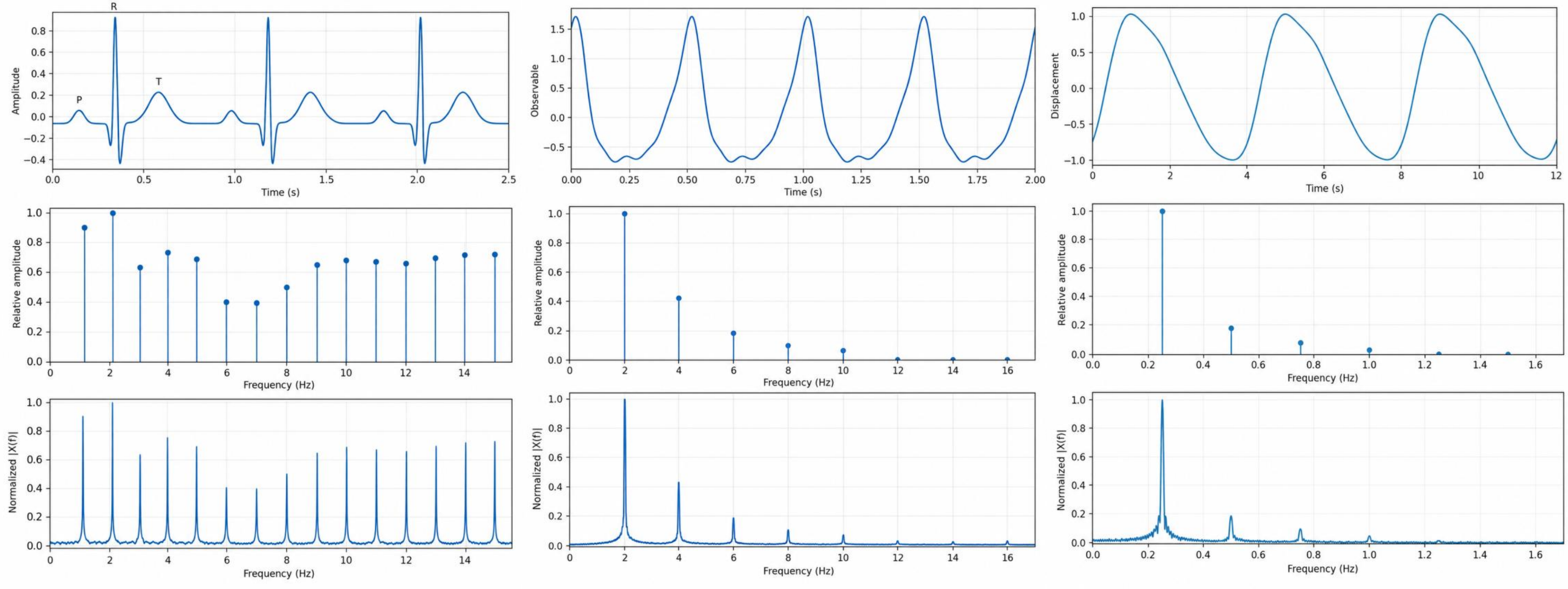
$$x(t) \leftrightarrow X(f)$$

A comparison of FS and CTFT

Time-domain

FS

CTFT (w/ noise)



Heartbeat | Breathing | Wheel spinning

CTFT give us **frequency domain spectrums:**

How a signal behaves across frequencies? Is it changing rapidly? Is it smooth?

Question

- What is the Fourier Transform of a cosine wave?

$$x(t) = \cos 2\pi f_0 t$$

$$X(f) = ?$$

Question

- What is the Fourier Transform of a cosine wave?

Pulse function
(Dirac delta function)

$$\delta(f) = \begin{cases} 0, f \neq 0 \\ \infty, f = 0 \end{cases},$$

$$\int_{-\infty}^{\infty} \delta(f) df = 1.$$

$$x(t) = \cos 2\pi f_0 t$$

$$X(f) = \pi\delta(f - f_0) + \pi\delta(f + f_0)$$

- A pulse at frequency f_0 and a pulse at $-f_0$.
- Why there are negative frequencies?

Convolution

- Convolution is the **dual operation** for multiplication in Fourier Analysis.

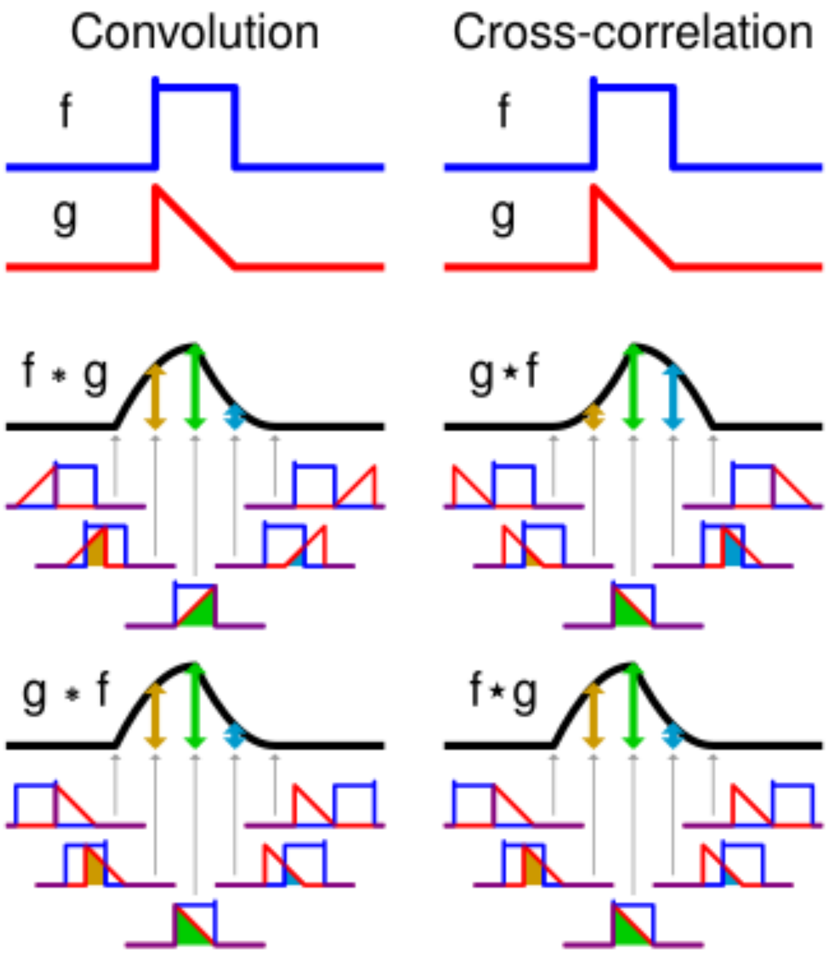
$$h(t) * x(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\mathrm{d}\tau$$

$$x(t) * h(t) \leftrightarrow X(f) \cdot H(f)$$

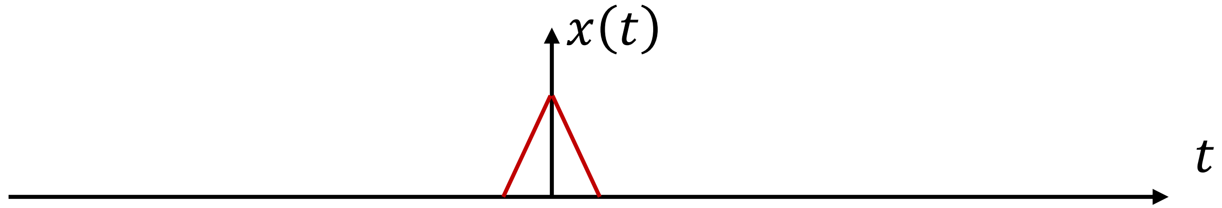
$$x(t) \cdot h(t) \leftrightarrow X(f) * H(f)$$

An important note: Convolution and cross-correlation

- Similarity: fix one signal, move the other, calculate the area of intersection.
- Difference:
  - Convolution: flip one signal. The operation is **commutative**.
  - Cross-correlation: do not flip signal. The operation is **not commutative**.

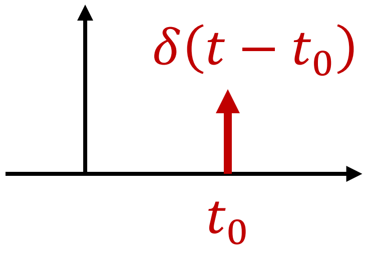


Example convolutions (time-domain)

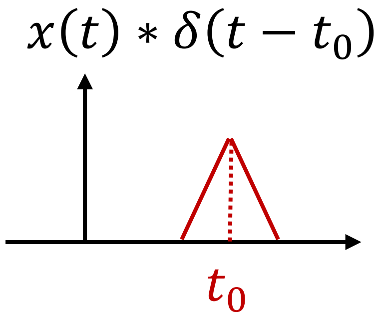


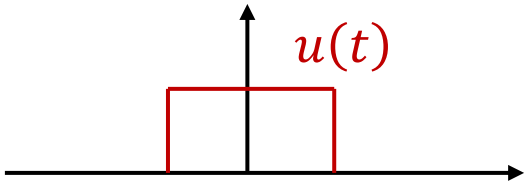
$$sinc(t) = \frac{\sin \pi t}{\pi t}$$

$*$

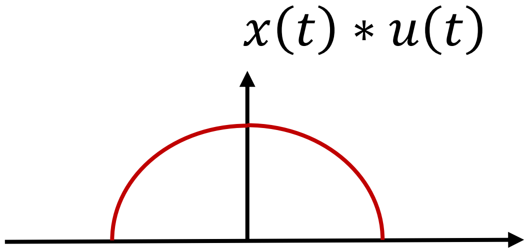


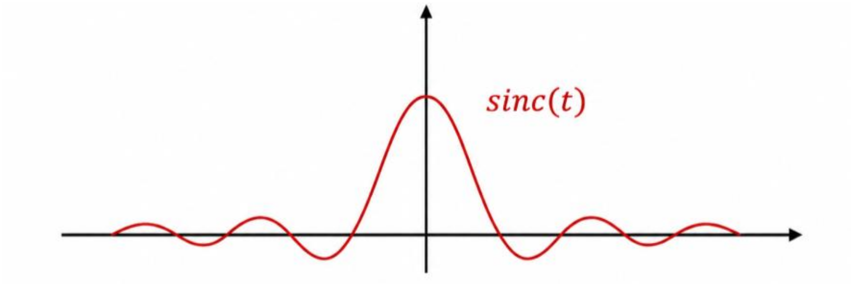
A pulse wave
Time shift the signal



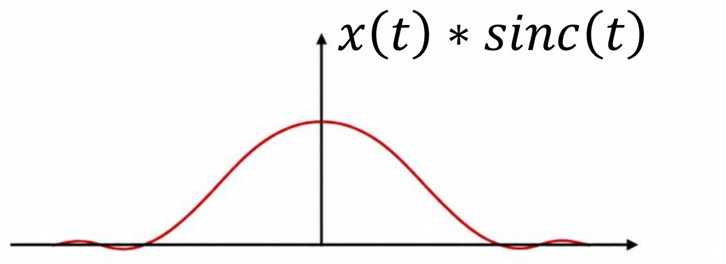


A square wave
Time-average the signal

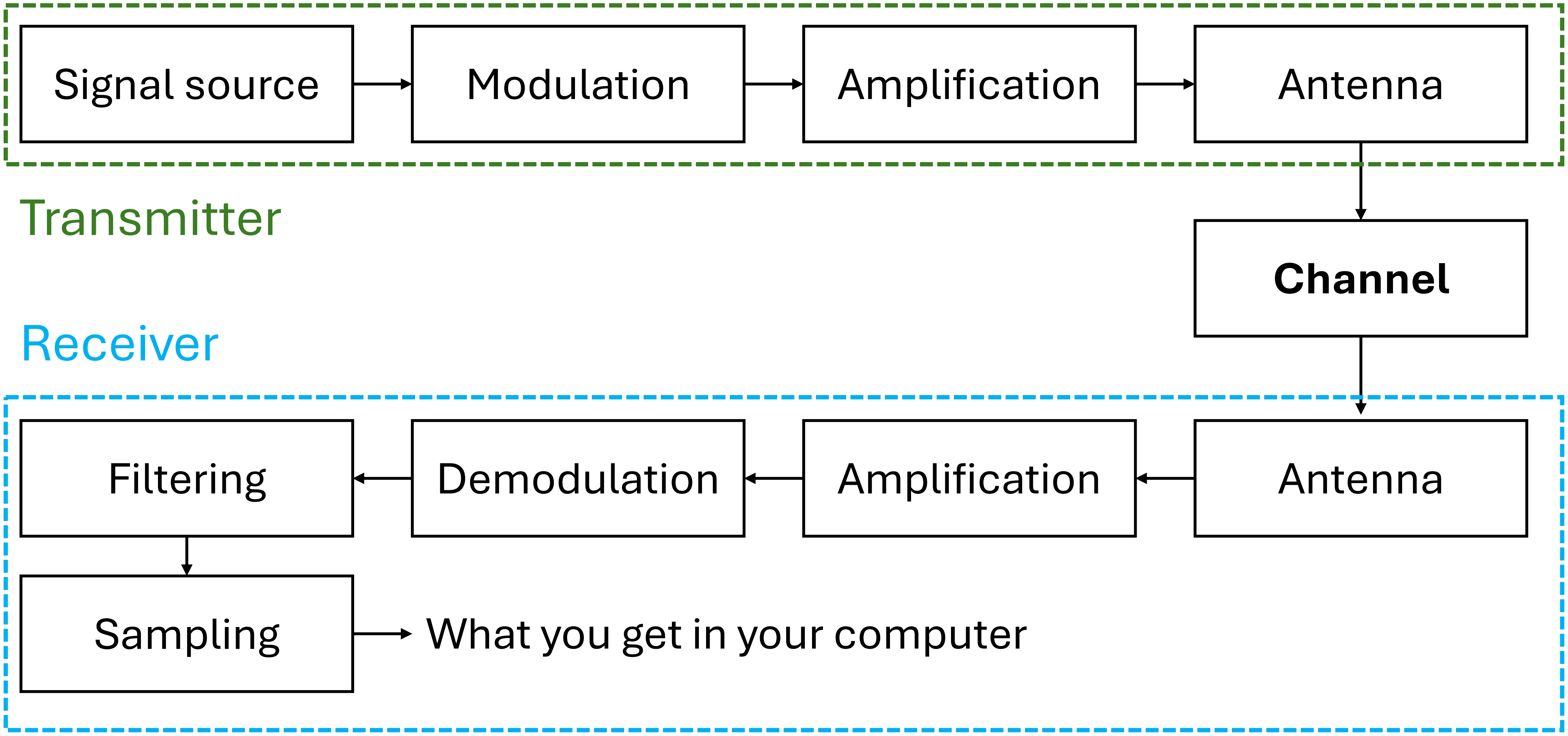




A sinc wave
Low-pass filter



A wireless system diagram



A wireless **channel**

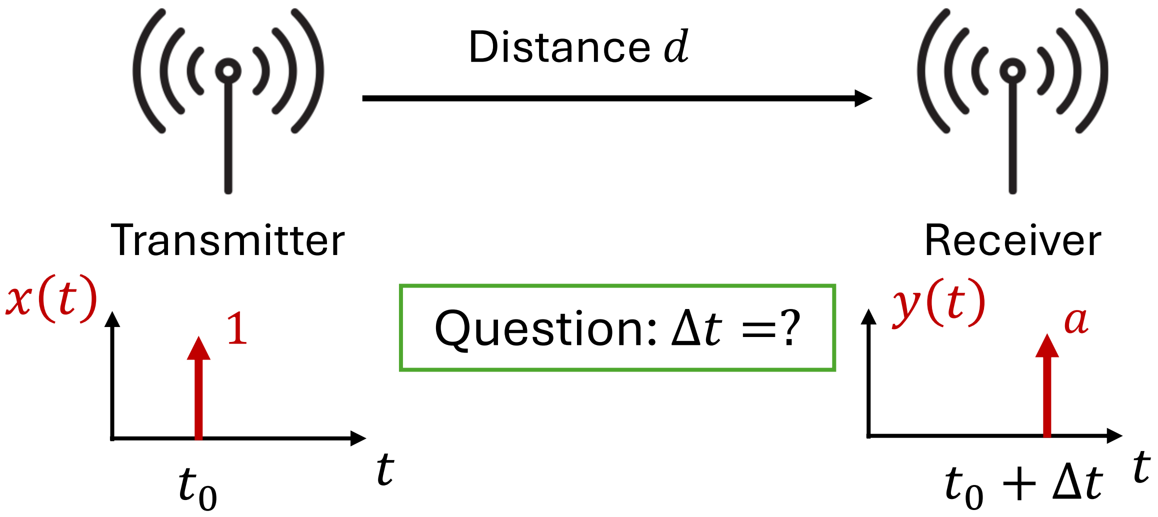
- A wireless channel describes how the signal goes from the transmitter to the receiver.
- Usually denoted as h.
- Channel is **convoluted in time** and **multiplied in frequency**.

$$y(t) = x(t) * h(t) + n(t)$$

$$Y(f) = X(f) \cdot H(f) + N(f)$$

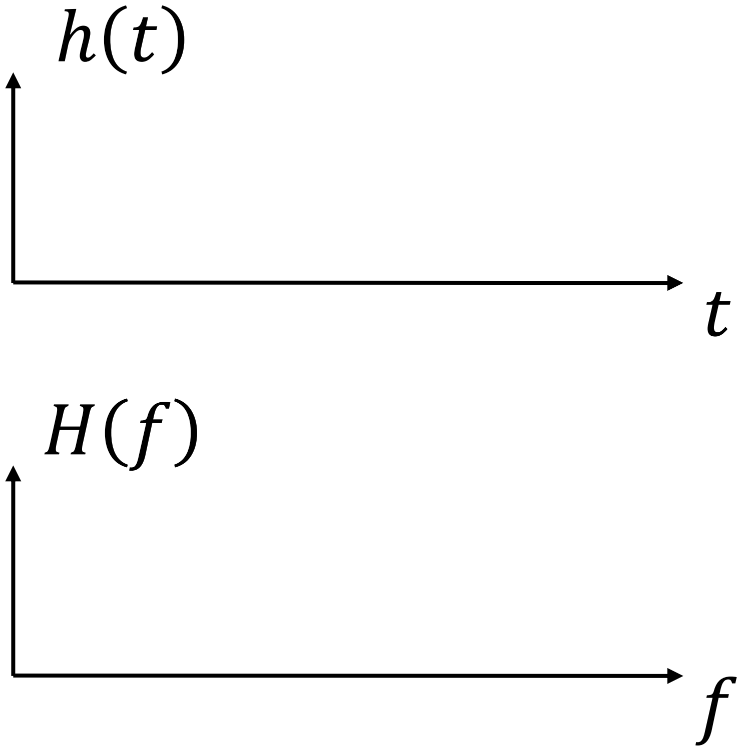
noise

A one-tap channel

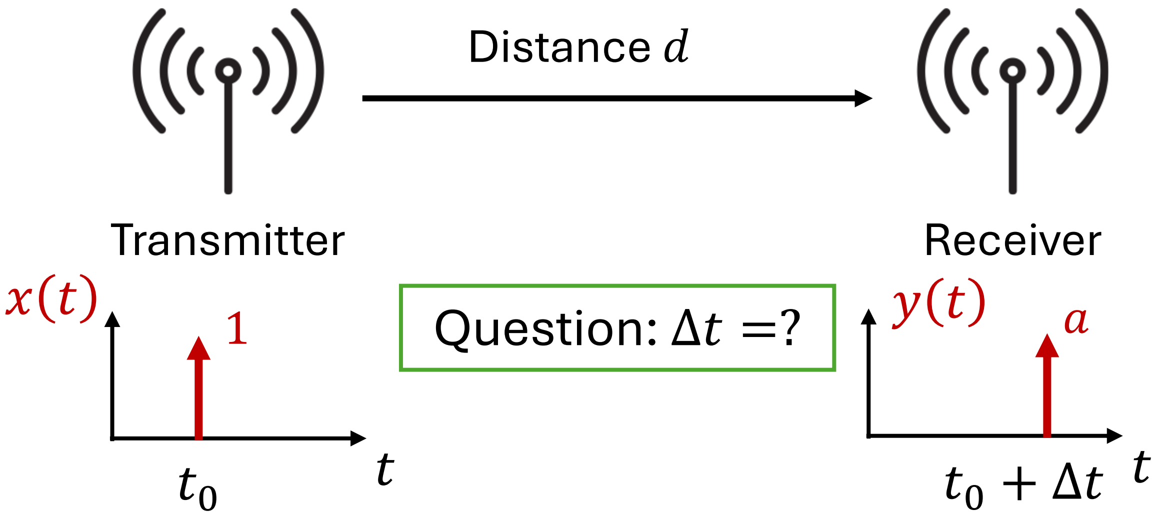


$$y(t) = x(t) * h(t)$$

$$Y(f) = X(f) \cdot H(f)$$

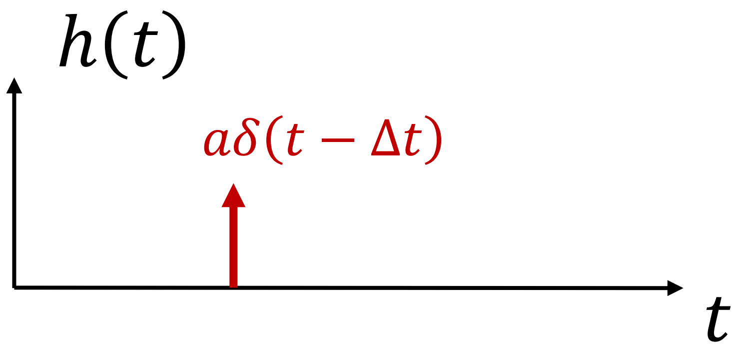


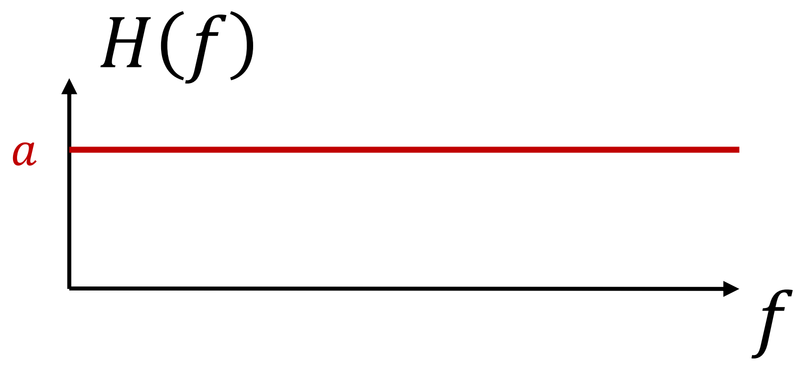
A **one-tap channel**



$$y(t) = x(t) * h(t)$$

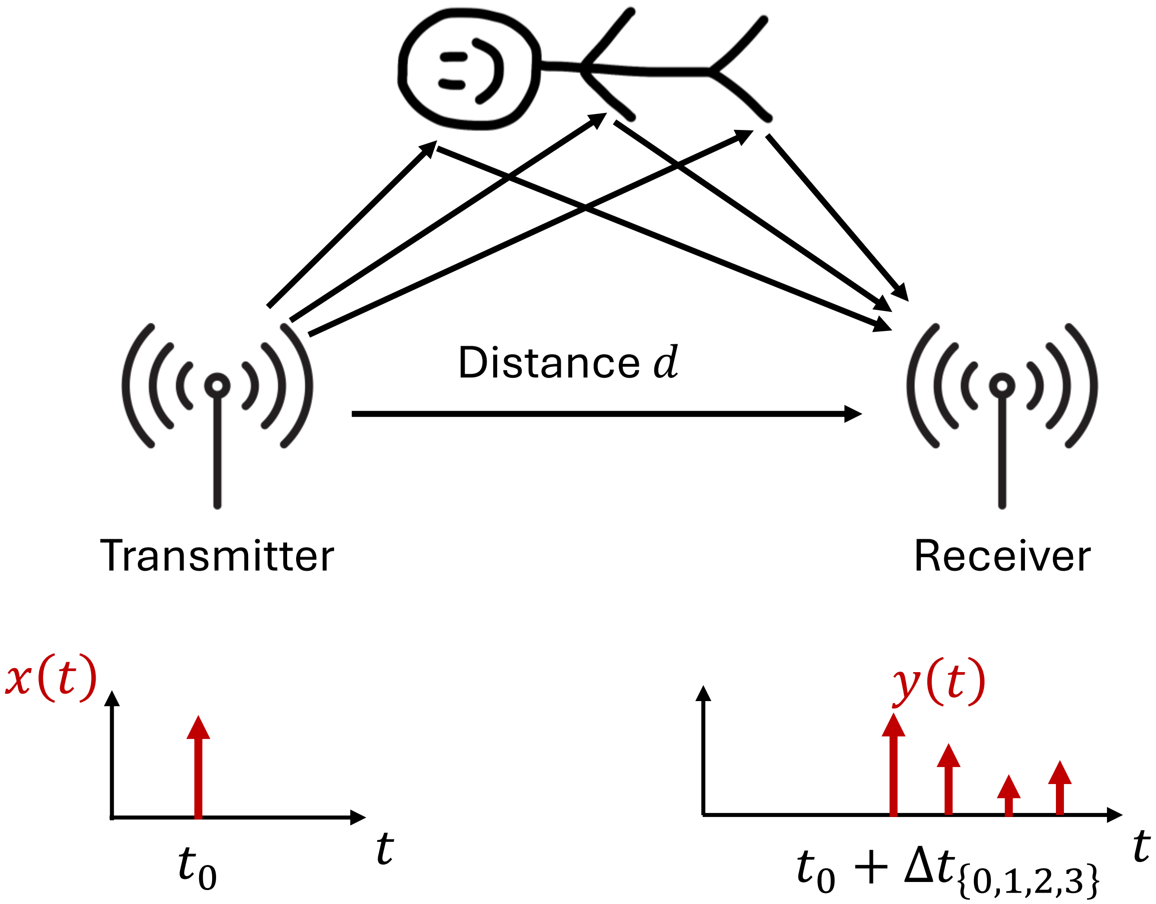
$$Y(f) = X(f) \cdot H(f)$$

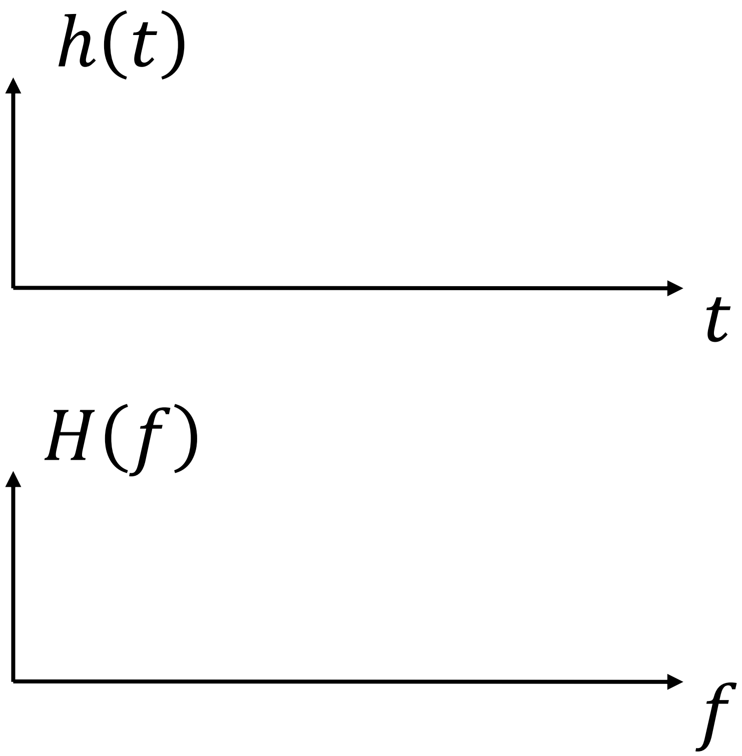




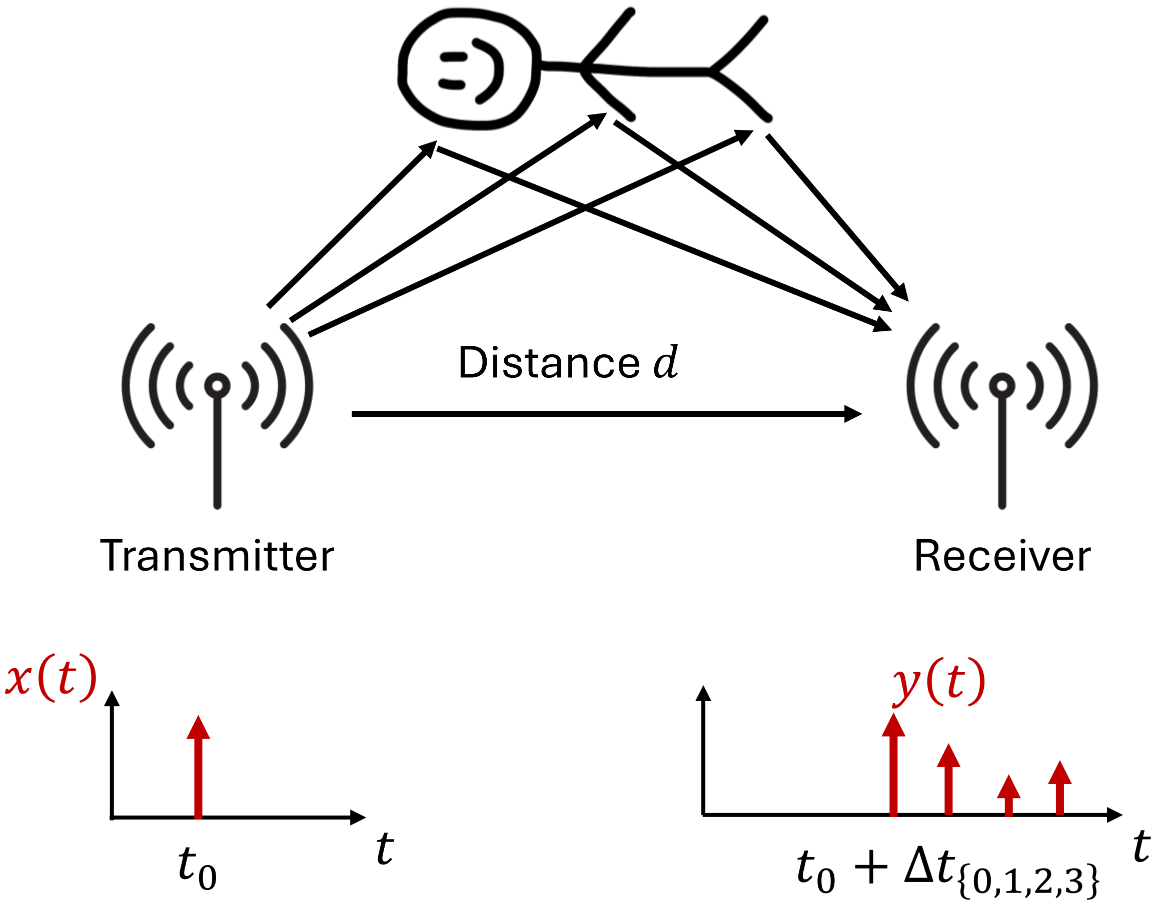
The channel is **not frequency-selective**.

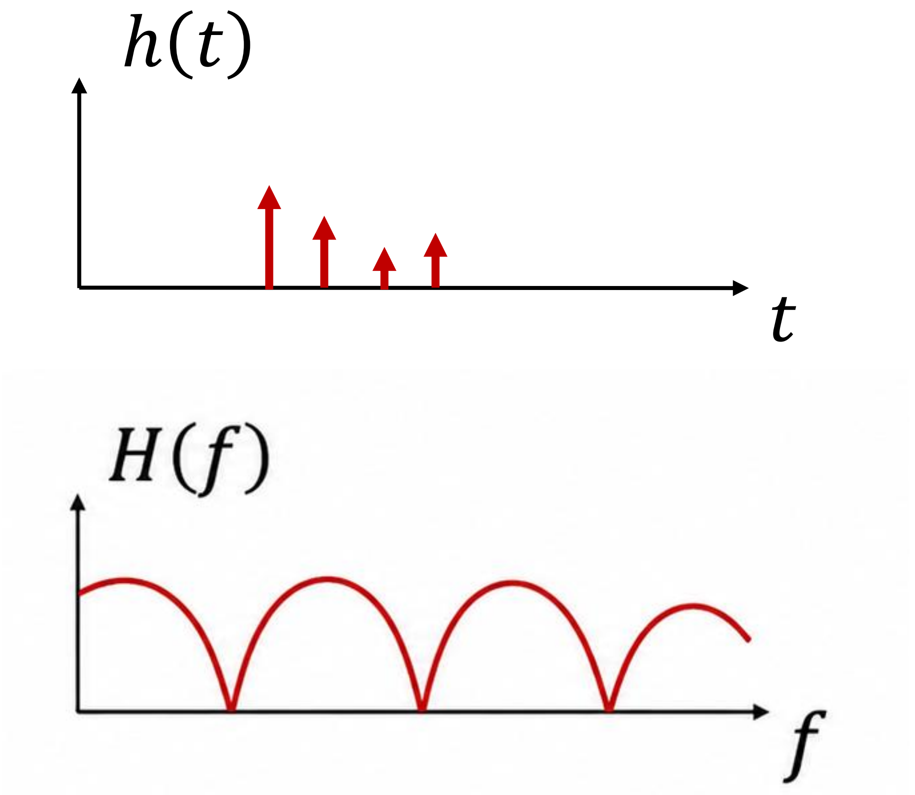
A **multi-tap channel**





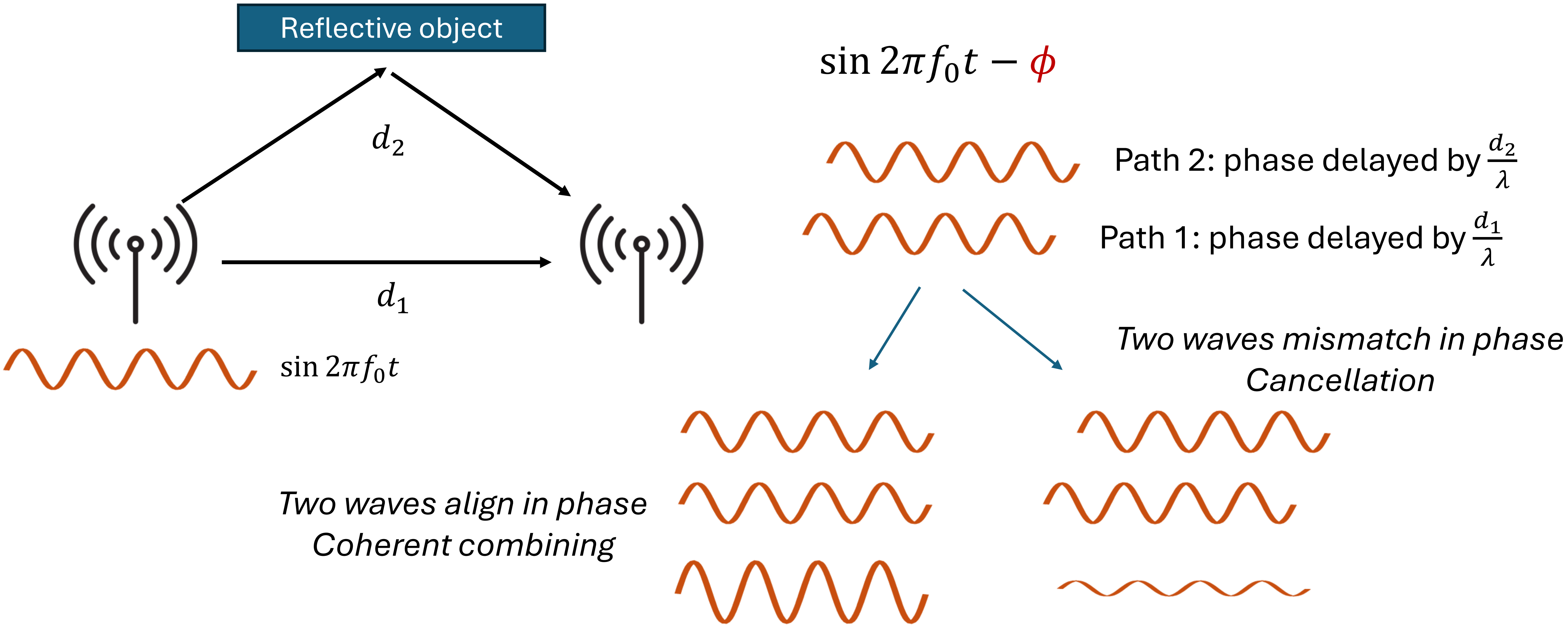
A multi-tap channel



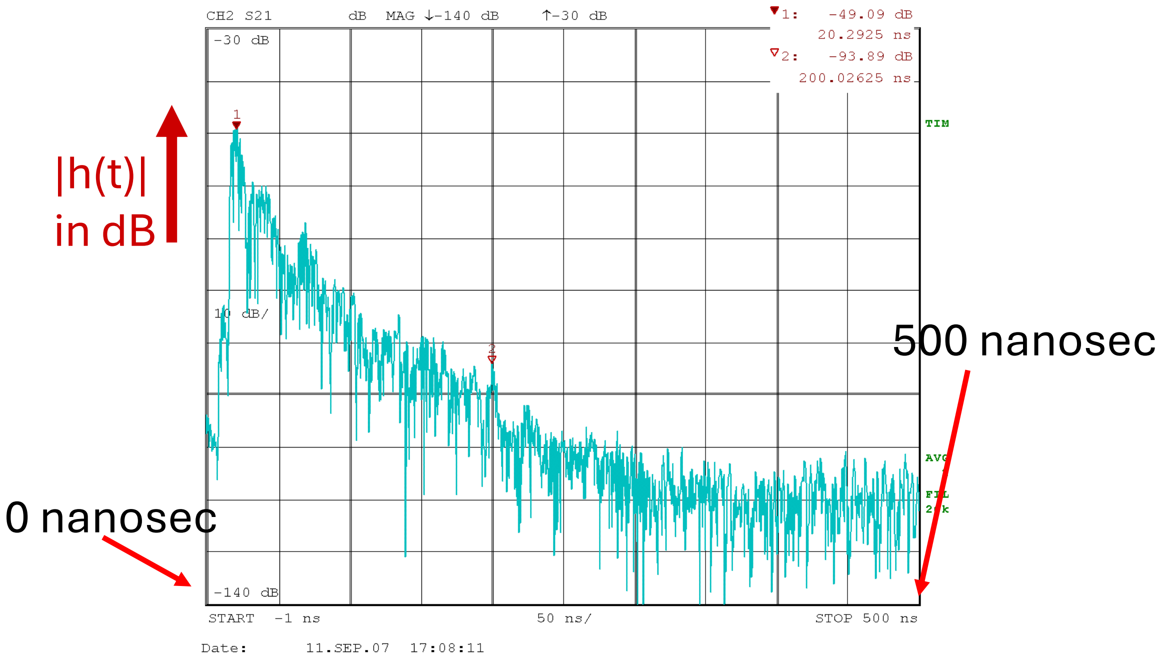


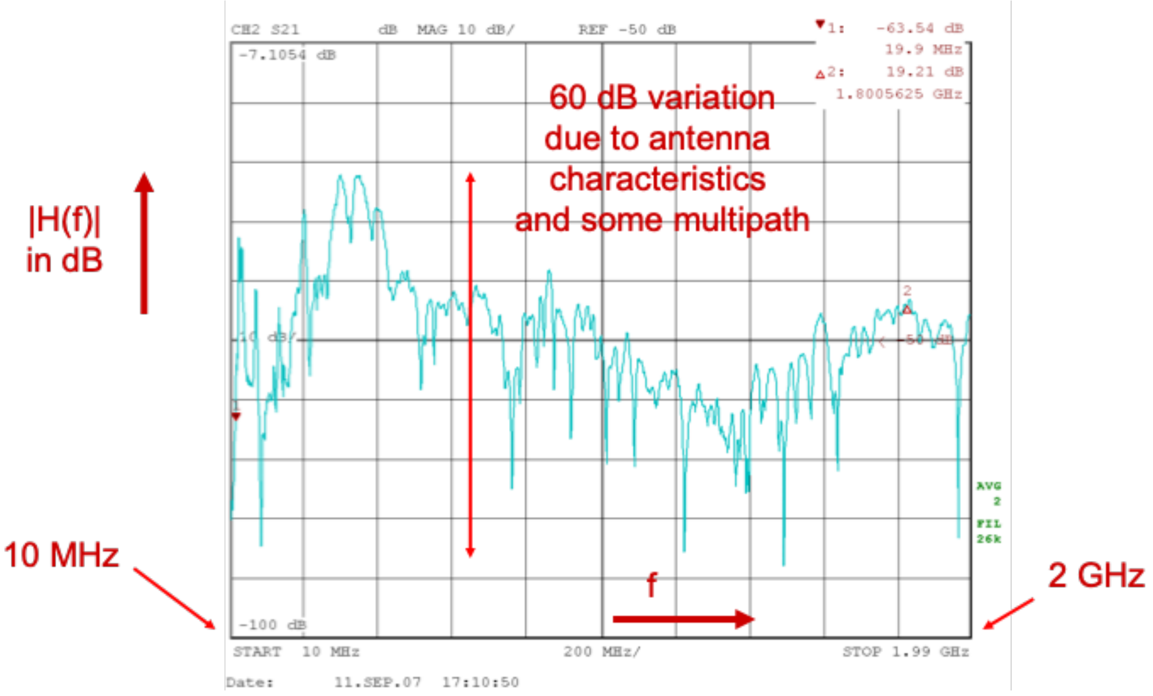
The channel is **frequency-selective**.

Where does frequency selectivity come from?



A measured channel





From 18-758 Wireless Communications. Picture credit: Prof. Rohit Negi.

Channel measurement

- **FMCW radars**: Send $x(t)$ of different frequencies, and measure the corresponding $H(f)$ values.
- **Pulse radars**: Send one-tap $x(t)$ and get the time-response $h(t)$.
- **Modern communication systems** (WiFi, cellular, etc.): Send a pilot sequence $x_0(t)$ and solve $h(t)$ according to $y_0(t)$.

- Communication system relies on accurate channel to recover the message from the received signal.
- Sensing system (this course) relies on **channel to acquire multipath and object information**.

Signal strength, signal power, and signal-to-noise ratio

$x(t) = Ae^{j2\pi f_0 t}$ A is signal strength (in volt).

$$P = \frac{V^2}{R} = \frac{|x(t)|^2}{R} = \frac{A^2}{R}$$ P is signal power (in watt).

In wireless systems, we usually deal with very small voltage/power,
So we use logarithmic metrics (dBV/dBW/dBm).

$$\mathrm{dBV}[x(t)] = 20\log_{10} A_{[V]}$$

$$\mathrm{dBW}[x(t)] = 10\log_{10} P_{[\mathrm{W}]}$$

$$\mathrm{dBm}[x(t)] = 10\log_{10} P_{[\mathrm{mW}]} = \mathrm{dBW}[x(t)] + 30$$

1W=0dBW=30dBm

1mW=-30dBW=0dBm

1μW=-60dBW=-30dBm

2mW≈3dBm 4mW≈6dBm

Signal-to-Noise Ratio (SNR)

Here we show an advantage of using dB systems:

- Assume we have a signal power of 1 mW = 0 dBm
- Noise is usually dependent on temperature.
- At room temperature, $\overline{n(t)^2} \approx -78\ \text{dBm} = 1.58 \times 10^{-8}$ mW

To calculate SNR, we use:

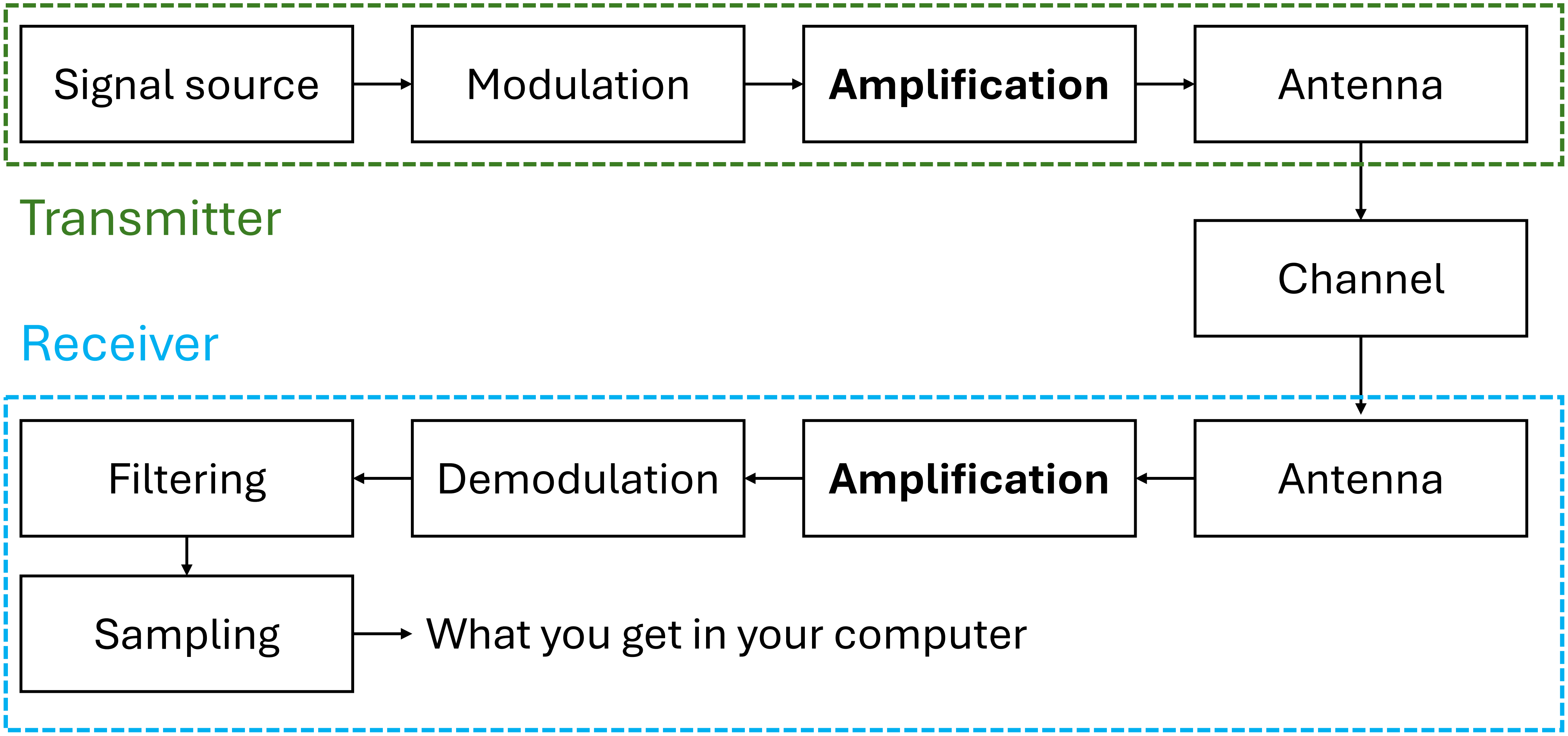
$$\text{SNR} = \frac{\overline{x(t)^2}}{\overline{n(t)^2}} = \frac{1\ \text{mW}}{1.58 \times 10^{-8}\ \text{mW}} = 6.3 \times 10^{7}$$

Or

$$\text{SNR} = \text{dBm}\left[\overline{x(t)^2}\right] - \text{dBm}\left[\overline{n(t)^2}\right] = 78\ \text{dB}$$

Much easier.

A wireless system diagram

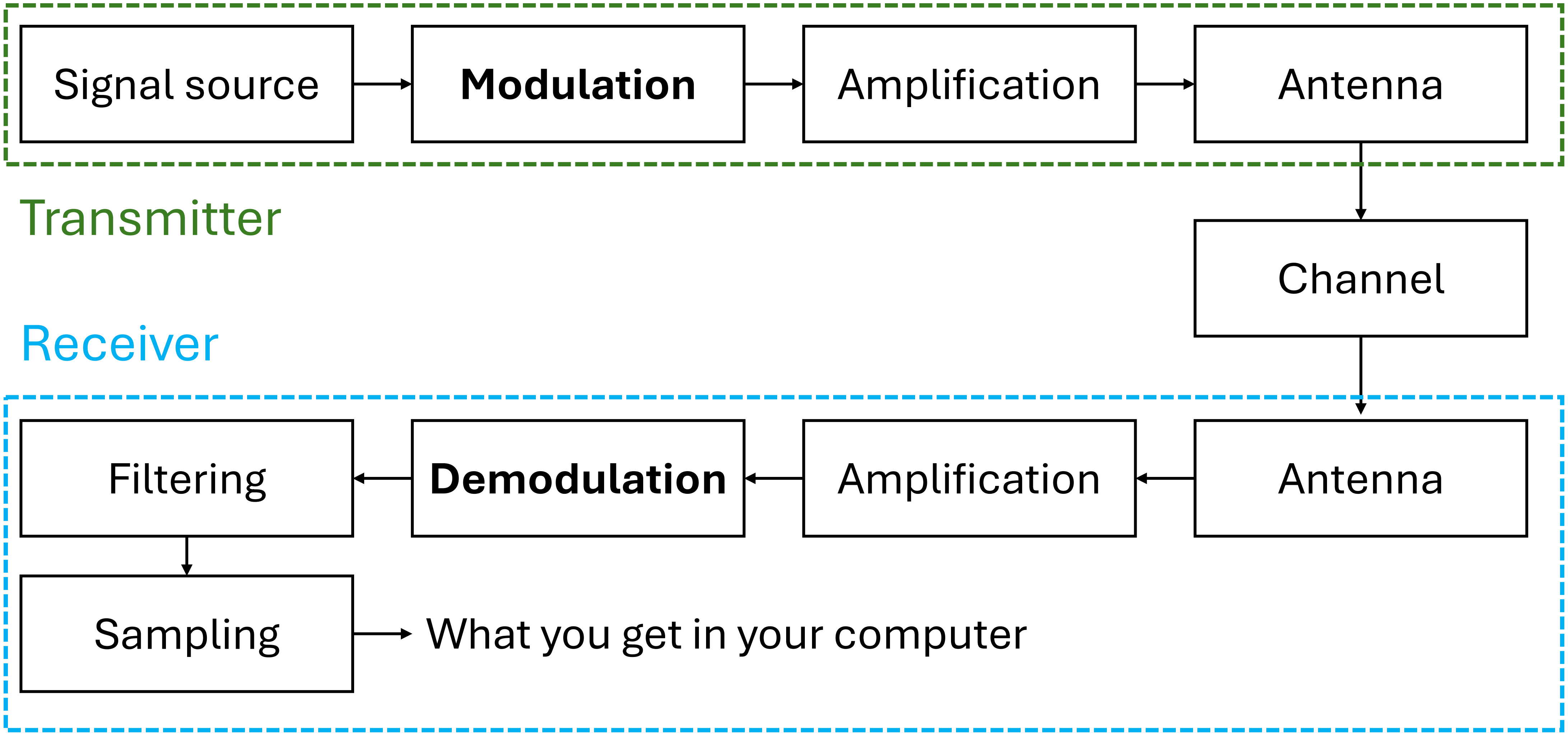


Amplification / Gain

- Amplification makes the wireless system work in longer range.
- Adaptive Gain Control (AGC) automatically adjusts amplification power according to signal power.
  - Usually exists in communication systems.
  - Need to be aware of if you sense with communication devices.

- TI AWR1843AOP does **NOT** have an AGC.
- You should adjust the gain according to the scenario that you are working with.

A wireless system diagram



Modulation: make EM carry information

Guglielmo Marconi
1874-1937

Signal (in volt)

Carrier frequency

$$s(t) = A(t)\boxed{\cos(2\pi f_c t + \phi)}$$

"Carrier"

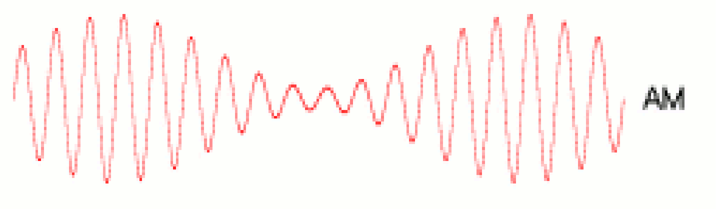
$$A(t) = \begin{cases} V, \text{key down} \\ 0, \text{key up} \end{cases}$$

Whether a key is tapped

The 0-1 code invented for this: Morse code.

Amplitude Modulation

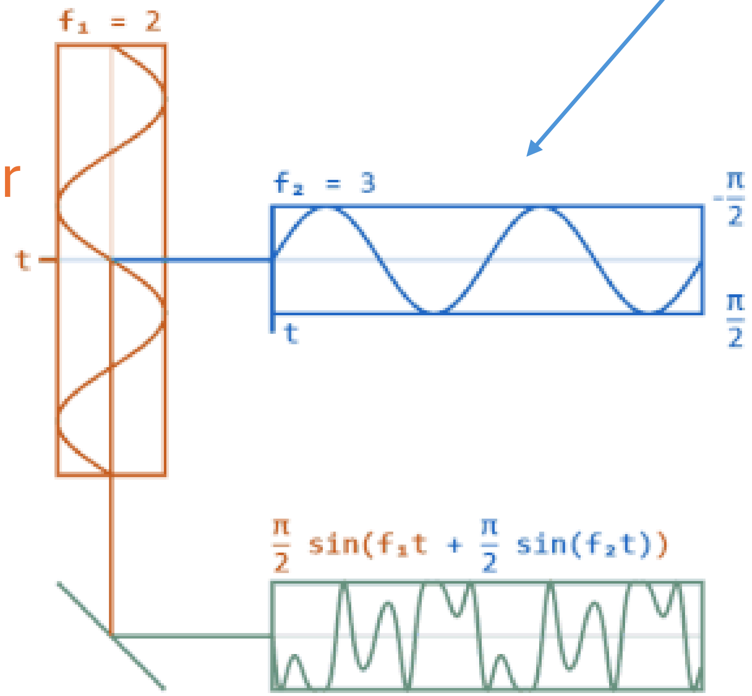
$$s(t) = A(t)\cos(2\pi f_c t + \phi)$$



Phase Modulation

$$s(t) = A\cos\left(2\pi f_c t + \phi(t)\right)$$

carrier



$s(t)$

Frequency Modulation

$$s(t) = A\cos 2\pi[f_c + f(t)]t + \phi$$